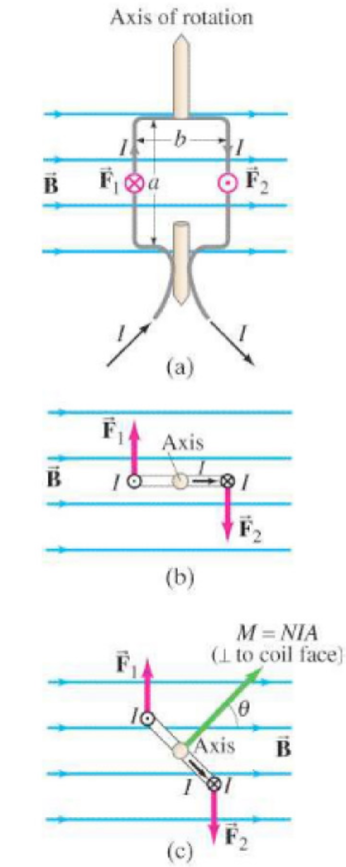

FIGURE 20–32 Calculating the torque on a current loop in a magnetic field $\vec{\mathbf{B}}$. (a) Loop face parallel to $\vec{\mathbf{B}}$ field lines; (b) top view; (c) loop makes an angle to $\vec{\mathbf{B}}$, reducing the torque since the lever arm is reduced.

* 20–9 Torque on a Current Loop; Magnetic Moment

When an electric current flows in a closed loop of wire placed in an external magnetic field, as shown in Fig. 20–32, the magnetic force on the current can produce a torque. This is the principle behind a number of important practical devices, including voltmeters, ammeters, and motors. (We discuss these applications in the next Section.) The interaction between a current and a magnetic field is important in other areas as well, including atomic physics.

Current flows through the loop in Fig. 20–32a, whose face we assume is parallel to $\vec{\mathbf{B}}$ and is rectangular. $\vec{\mathbf{B}}$ exerts no force and no torque on the horizontal segments of wire because they are parallel to the field and $\sin\theta = 0$ in Eq. 20–1. But the magnetic field does exert a force on each of the vertical sections of wire as shown, $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ (see also top view, Fig. 20–32b). By right-hand-rule-2 (Fig. 20–11c) the direction of the force on the upward current on the left is in the opposite direction from the equal magnitude force $\vec{\mathbf{F}}_2$ on the descending current on the right. These forces give rise to a net torque that tends to rotate the coil about its vertical axis.

Let us calculate the magnitude of this torque. From Eq. 20–2 (current $\perp \vec{\mathbf{B}}$), the force $F = IaB$, where a is the length of the vertical arm of the coil. The lever arm for each force is $b/2$, where b is the width of the coil and the "axis" is at the midpoint. The torques produced by $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ act in the same direction, so the total torque is the sum of the two torques:

$$\tau = IaB\frac{b}{2} + IaB\frac{b}{2} = IabB = IAB,$$

where $A = ab$ is the area of the coil. If the coil consists of N loops of wire, the current is then NI, so the torque becomes

$$\tau = NIAB.$$

If the coil makes an angle θ with the magnetic field, as shown in Fig. 20–32c, the forces are unchanged, but each lever arm is reduced from $\frac{1}{2}b$ to $\frac{1}{2}b\sin\theta$. Note that the angle θ is taken to be the angle between $\vec{\mathbf{B}}$ and the perpendicular to the face of the coil, Fig. 20–32c. So the torque becomes

Torque on current loop

$$\tau = NIAB\sin\theta. \qquad \textbf{(20–10)}$$

This formula, derived here for a rectangular coil, is valid for any shape of flat coil.

The quantity NIA is called the **magnetic dipole moment** of the coil:

Magnetic dipole moment

$$M = NIA \qquad \textbf{(20–11)}$$

and is considered a vector perpendicular to the coil.

EXAMPLE 20–12 Torque on a coil. A circular coil of wire has a diameter of 20.0 cm and contains 10 loops. The current in each loop is 3.00 A, and the coil is placed in a 2.00-T external magnetic field. Determine the maximum and minimum torque exerted on the coil by the field.

APPROACH Equation 20–10 is valid for any shape of coil, including circular loops. Maximum and minimum torque are determined by the angle θ the coil makes with the magnetic field.

SOLUTION The area of one loop of the coil is

$$A = \pi r^2 = \pi(0.100\,\mathrm{m})^2 = 3.14 \times 10^{-2}\,\mathrm{m}^2.$$

The maximum torque occurs when the coil's face is parallel to the magnetic field, so $\theta = 90°$ in Fig. 20–32c, and $\sin\theta = 1$ in Eq. 20–10:

$$\tau = NIAB\sin\theta = (10)(3.00\,\mathrm{A})(3.14 \times 10^{-2}\,\mathrm{m}^2)(2.00\,\mathrm{T})(1) = 1.88\,\mathrm{N\cdot m}.$$

The minimum torque occurs if $\sin\theta = 0$, for which $\theta = 0°$, and then $\tau = 0$ from Eq. 20–10.

NOTE If the coil is free to turn, it will rotate toward the orientation with $\theta = 0°$.