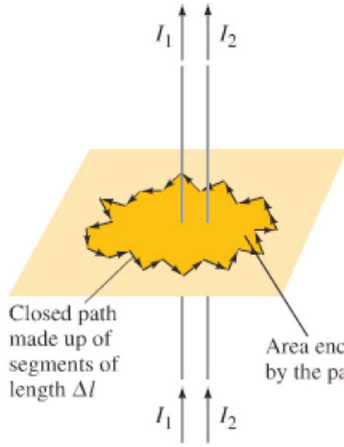


FIGURE 20–28 Arbitrary path enclosing electric currents, for Ampère's law. The path is broken down into segments of equal length Δl. The total current enclosed by the path shown is $I_{encl} = I_1 + I_2$.

* 20–8 Ampère's Law

In Section 20–5 we saw that Eq. 20–6 gives the relation between the current in a long straight wire and the magnetic field it produces. This equation is valid *only* for a long straight wire. Is there a general relation between a current in a wire of any shape and the magnetic field around it? Yes: the French scientist André Marie Ampère (1775–1836) proposed such a relation shortly after Oersted's discovery. Consider any (arbitrary) closed path around a current, as shown in Fig. 20–28, and imagine this path as being made up of short segments each of length Δl. We take the product of the length of each segment times the component of magnetic field $\vec{\mathbf{B}}$ parallel to that segment. If we now sum all these terms, the result (said Ampère) will be equal to μ_0 times the net current I_{encl} that passes through the surface *enclosed* by the path. This is known as **Ampère's law** and can be written

AMPÈRE'S LAW

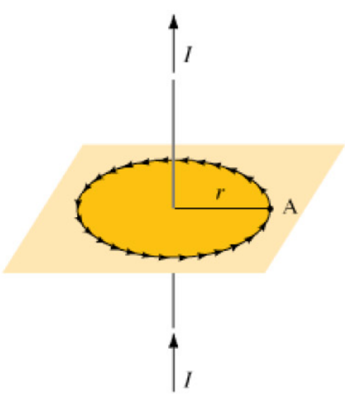
$$\Sigma B_{\parallel} \Delta l = \mu_0 I_{encl}. \qquad \textbf{(20–9)}$$

The symbol Σ means "the sum of" and $B_{\parallel}$ means the component of $\vec{\mathbf{B}}$ parallel to that particular Δl. The lengths Δl are chosen small enough so that $B_{\parallel}$ is essentially constant on each length. The sum must be made over a closed path, and I_{encl} is the total net current enclosed by the closed path.

* Field Due to a Straight Wire

We can check Ampère's law by applying it to the simple case of a long straight wire carrying a current I. Let us find the magnitude of B at point A, a distance r from the wire in Fig. 20–29. The magnetic field lines are circles with the wire at their center (as in Fig. 20–8). As the path to be used in Eq. 20–9, we choose a convenient one: a circle of radius r, because at any point on this path, $\vec{\mathbf{B}}$ will be tangent to this circle. For any short segment of the circle (Fig. 20–29), $\vec{\mathbf{B}}$ will be parallel to that segment, so $B_{\parallel} = B$. Suppose we break the circular path down into 100 segments.[†] Then Ampère's law states that

FIGURE 20–29 Circular path of radius r.



$$(B\,\Delta l)_1 + (B\,\Delta l)_2 + (B\,\Delta l)_3 + \cdots + (B\,\Delta l)_{100} = \mu_0 I.$$

The dots represent all the terms we did not write down. All the segments are the same distance from the wire, so by symmetry we expect B to be the same at each segment. We can then factor out B from the sum:

$$B(\Delta l_1 + \Delta l_2 + \Delta l_3 + \cdots + \Delta l_{100}) = \mu_0 I.$$

The sum of the segment lengths Δl is just the circumference of the circle, $2\pi r$.

[†] Actually, Ampère's law is precisely accurate when there is an infinite number of infinitesimally short segments, but that leads into calculus.