We can write a similar formula for the charge $Q\,(= CV_C)$ on the capacitor:

$$Q = Q_0\left(1 - e^{-t/RC}\right),$$

where Q_0 represents the maximum charge.

The product of the resistance R times the capacitance C, which appears in the exponent, is called the **time constant** τ of the circuit:

Time constant $\tau = RC$

$$\tau = RC. \qquad \textbf{(19–7)}$$

The time constant is a measure of how quickly the capacitor becomes charged. [The units of RC are $\Omega \cdot \mathrm{F} = (\mathrm{V/A})(\mathrm{C/V}) = \mathrm{C/(C/s)} = \mathrm{s}$.] Specifically, it can be shown that the product RC gives the time required for the capacitor's voltage (and charge) to reach 63% of the maximum. This can be checked[†] using any calculator with an e^x key: $e^{-1} = 0.37$, so for $t = RC$, then $\left(1 - e^{-t/RC}\right) = \left(1 - e^{-1}\right) = (1 - 0.37) = 0.63$. In a circuit, for example, where $R = 200\,\mathrm{k}\Omega$ and $C = 3.0\,\mu\mathrm{F}$, the time constant is $\left(2.0 \times 10^5\,\Omega\right)\left(3.0 \times 10^{-6}\,\mathrm{F}\right) = 0.60\,\mathrm{s}$. If the resistance is much smaller, the time constant is much smaller and the capacitor becomes charged almost instantly. This makes sense, since a lower resistance will retard the flow of charge less. All circuits contain some resistance (if only in the connecting wires), so a capacitor can never be charged instantaneously when connected to a battery.

Capacitor discharges

The circuit just discussed involved the *charging* of a capacitor by a battery through a resistance. Now let us look at another situation: a capacitor is already charged (say, to a voltage V_0 and charge Q_0), and it is then allowed to *discharge* through a resistance R as shown in Fig. 19–21a. (In this case there is no battery.) When the switch S is closed, charge begins to flow through resistor R from one side of the capacitor toward the other side, until it is fully discharged. The voltage across the capacitor decreases, as shown in Fig. 19–21b. This "exponential decay" curve is given by

$$V_C = V_0 e^{-t/RC},$$

where V_0 is the initial voltage across the capacitor. The voltage falls 63% of the way to zero (to $0.37V_0$) in a time $\tau = RC$. Because the charge Q on the capacitor is $Q = CV$, we can write

$$Q = Q_0 e^{-t/RC}$$

for a discharging capacitor, where Q_0 is the initial charge.

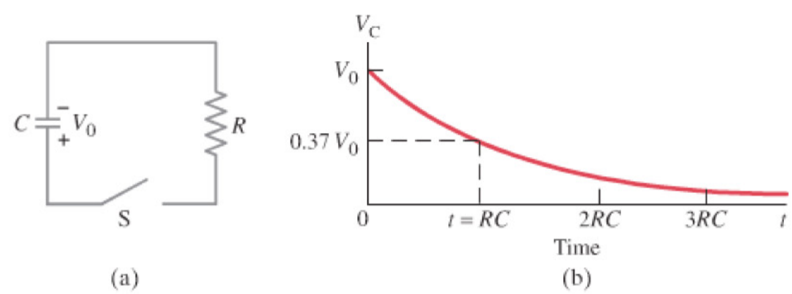

FIGURE 19–21 For the RC circuit shown in (a), the voltage V_C on the capacitor decreases with time, as shown in (b), after the switch S is closed. The charge on the capacitor follows the same curve since $Q \propto V$.

EXAMPLE 19–12 A discharging *RC* circuit. If a charged capacitor, $C = 35\,\mu\mathrm{F}$, is connected to a resistance $R = 120\,\Omega$ as in Fig. 19–21a, how much time will elapse until the voltage falls to 10% of its original (maximum) value?

APPROACH The voltage across the capacitor decreases according to $V_C = V_0 e^{-t/RC}$. We set $V_C = 0.10V_0$ (10% of V_0), but first we need to calculate $\tau = RC$.

[†] More simply, since $e = 2.718\cdots$, then $e^{-1} = 1/e = 1/2.718 = 0.37$. Note that e is the inverse operation to the natural logarithm ln: $\ln(e) = 1$, and $\ln\left(e^x\right) = x$.