**EXAMPLE 18–15** **ESTIMATE** **Capacitance of an axon.** (*a*) Do an order-of-magnitude estimate for the capacitance of an axon 10 cm long of radius 10 $\mu$m. The thickness of the membrane is about $10^{-8}$ m, and the dielectric constant is about 3. (*b*) By what factor does the concentration (number of ions per volume) of $Na^+$ ions in the cell change as a result of one action potential?

**APPROACH** We model the membrane of an axon as a cylindrically shaped parallel-plate capacitor, with opposite charges on each side. The separation of the "plates" is the thickness of the membrane, $d \approx 10^{-8}$ m. We first calculate the area of the cylinder and then can use Eq. 17–9, $C = K\epsilon_0 A/d$, to find the capacitance. In (*b*), we use the voltage change during one action potential to find the amount of charge moved across the membrane.

**SOLUTION** (*a*) The area $A$ is the area of a cylinder of radius $r$ and length $l$:

$$A = 2\pi r l \approx (6.28)(10^{-5}\,\mathrm{m})(0.1\,\mathrm{m}) \approx 6 \times 10^{-6}\,\mathrm{m}^2.$$

From Eq. 17–9, we have

$$C = K\epsilon_0 \frac{A}{d} \approx (3)(8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2})\frac{6 \times 10^{-6}\,\mathrm{m}^2}{10^{-8}\,\mathrm{m}} \approx 10^{-8}\,\mathrm{F}.$$

(*b*) Since the voltage changes from $-70$ mV to about $+30$ mV, the total change is about 100 mV. The amount of charge that moves is then

$$Q = CV \approx (10^{-8}\,\mathrm{F})(0.1\,\mathrm{V}) = 10^{-9}\,\mathrm{C}.$$

Each ion carries a charge $e = 1.6 \times 10^{-19}\,\mathrm{C}$, so the number of ions that flow per action potential is $Q/e = (10^{-9}\,\mathrm{C})/(1.6 \times 10^{-19}\,\mathrm{C}) \approx 10^{10}$. The volume of our cylindrical axon is

$$V = \pi r^2 l \approx (3)(10^{-5}\,\mathrm{m})^2(0.1\,\mathrm{m}) = 3 \times 10^{-11}\,\mathrm{m}^3,$$

and the concentration of $Na^+$ ions inside the cell (Table 18–2) is $15\,\mathrm{mol/m^3} = 15 \times 6.02 \times 10^{23}\,\mathrm{ions/m^3} \approx 10^{25}\,\mathrm{ions/m^3}$. Thus, the cell contains $(10^{25}\,\mathrm{ions/m^3}) \times (3 \times 10^{-11}\,\mathrm{m^3}) \approx 3 \times 10^{14}$ $Na^+$ ions. One action potential, then, will change the concentration of $Na^+$ ions by about $10^{10}/(3 \times 10^{14}) = \frac{1}{3} \times 10^{-4}$, or 1 part in 30,000. This tiny change would not be measurable.

Thus, even 1000 action potentials will not alter the concentration significantly. The sodium pump does not, therefore, have to remove $Na^+$ ions quickly after an action potential, but can operate slowly over time to maintain a relatively constant concentration.

The propagation of a nerve pulse as described here applies to an unmyelinated axon. Myelinated axons, on the other hand, are insulated from the extracellular fluid by the myelin sheath except at the nodes of Ranvier (see Fig. 18–27). An action potential cannot be generated where there is a myelin sheath. Once such a neuron is stimulated, the pulse will still travel along the membrane, but there is resistance and the pulse becomes smaller as it moves down the axon. Nonetheless, the weakened signal can still stimulate a full-fledged action potential when it reaches a node of Ranvier. Thus, the signal is repeatedly amplified at these points. Compare this to an unmyelinated neuron, in which the signal is continually amplified by repeated action potentials all along its length, requiring much more energy. Development of myelinated neurons can be seen as a significant evolutionary step, for it meant reliable transmission of nerve pulses with less energy expended. And the pulses travel more quickly, since ordinary conduction is faster than the repeated production of action potentials, whose speed depends on the flow of ions across the membrane.