A vector $\vec{\mathbf{V}}$ can be multiplied by a scalar c. We define their product so that $c\vec{\mathbf{V}}$ has the same direction as $\vec{\mathbf{V}}$ and has magnitude cV. That is, multiplication of a vector by a positive scalar c changes the magnitude of the vector by a factor c but doesn't alter the direction. If c is a negative scalar, the magnitude of the product $c\vec{\mathbf{V}}$ is still cV (without the minus sign), but the direction is precisely opposite to that of $\vec{\mathbf{V}}$. See Fig. 3–9.

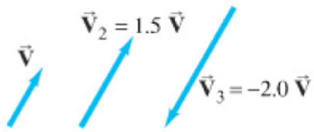

FIGURE 3–9 Multiplying a vector $\vec{\mathbf{V}}$ by a scalar c gives a vector whose magnitude is c times greater and in the same direction as $\vec{\mathbf{V}}$ (or opposite direction if c is negative).

3–4 Adding Vectors by Components

Adding vectors graphically using a ruler and protractor is often not sufficiently accurate and is not useful for vectors in three dimensions. We discuss now a more powerful and precise method for adding vectors. But do not forget graphical methods—they are always useful for visualizing, for checking your math, and thus for getting the correct result.

Consider first a vector $\vec{\mathbf{V}}$ that lies in a particular plane. It can be expressed as the sum of two other vectors, called the **components** of the original vector. The components are usually chosen to be along two perpendicular directions. The process of finding the components is known as **resolving the vector into its components**. An example is shown in Fig. 3–10; the vector $\vec{\mathbf{V}}$ could be a displacement vector that points at an angle $\theta = 30°$ north of east, where we have chosen the positive x axis to be to the east and the positive y axis north. This vector $\vec{\mathbf{V}}$ is resolved into its x and y components by drawing dashed lines out from the tip (A) of the vector (lines AB and AC) making them perpendicular to the x and y axes. Then the lines 0B and 0C represent the x and y components of $\vec{\mathbf{V}}$, respectively, as shown in Fig. 3–10b. These **vector components** are written $\vec{\mathbf{V}}_x$ and $\vec{\mathbf{V}}_y$. We generally show vector components as arrows, like vectors, but dashed. The *scalar components*, V_x and V_y, are numbers, with units, that are given a positive or negative sign depending on whether they point along the positive or negative x or y axis. As can be seen in Fig. 3–10, $\vec{\mathbf{V}}_x + \vec{\mathbf{V}}_y = \vec{\mathbf{V}}$ by the parallelogram method of adding vectors.

Resolving a vector into components

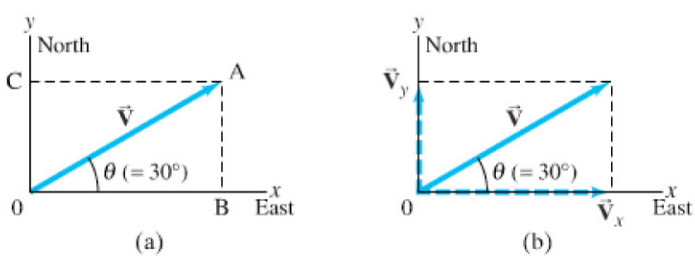

FIGURE 3–10 Resolving a vector $\vec{\mathbf{V}}$ into its components along an arbitrarily chosen set of x and y axes. The components, once found, themselves represent the vector. That is, the components contain as much information as the vector itself.

Space is made up of three dimensions, and sometimes it is necessary to resolve a vector into components along three mutually perpendicular directions. In rectangular coordinates the components are $\vec{\mathbf{V}}_x$, $\vec{\mathbf{V}}_y$, and $\vec{\mathbf{V}}_z$. Resolution of a vector in three dimensions is merely an extension of the above technique. We will mainly be concerned with situations in which the vectors are in a plane and two components are all that are necessary.

To add vectors using the method of components, we need to use the trigonometric functions sine, cosine, and tangent, which we now review.