17–2 Relation between Electric Potential and Electric Field

The effects of any charge distribution can be described either in terms of electric field or in terms of electric potential. Electric potential is often easier to use since it is a scalar, whereas electric field is a vector. There is an intimate connection between the potential and the field. Let us consider the case of a uniform electric field, such as that between the parallel plates of Fig. 17–1 whose difference of potential is V_{ba}. The work done by the electric field to move a positive charge q from a to b is equal to the negative of the change in potential energy (Eq. 17–2b), so

$$W = -q(V_b - V_a) = -qV_{ba}.$$

We can also write the work done as the force times distance, where the force on q is $F = qE$, so

$$W = Fd = qEd,$$

where d is the distance (parallel to the field lines) between points a and b. We now set these two expressions for W equal and find $qV_{ba} = -qEd$, or

V related to uniform E

$$V_{ba} = -Ed. \qquad [E \text{ uniform}] \qquad \textbf{(17–4a)}$$

If we solve for E, we find

$$E = -\frac{V_{ba}}{d}. \qquad [E \text{ uniform}] \qquad \textbf{(17–4b)}$$

Units for E: 1 N/C = 1 V/m

From Eq. 17–4b we see that the units for electric field can be written as volts per meter (V/m) as well as newtons per coulomb (N/C). These are equivalent in general, since $1\ \text{N/C} = 1\ \text{N}\cdot\text{m/C}\cdot\text{m} = 1\ \text{J/C}\cdot\text{m} = 1\ \text{V/m}$. The minus sign in Eq. 17–4b tells us that $\vec{\mathbf{E}}$ points in the direction of decreasing potential V.

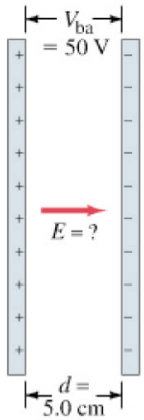

FIGURE 17–5 Example 17–3.

EXAMPLE 17–3 Electric field obtained from voltage. Two parallel plates are charged to produce a potential difference of 50 V. If the separation between the plates is 0.050 m, calculate the magnitude of the electric field in the space between the plates (Fig. 17–5).

APPROACH We apply Eq. 17–4b to obtain the magnitude of E, assumed uniform.

SOLUTION The magnitude of the electric field is

$$E = V_{ba}/d = (50\ \text{V}/0.050\ \text{m}) = 1000\ \text{V/m}.$$

NOTE Equations 17–4 apply only for a uniform electric field. The general relationship between $\vec{\mathbf{E}}$ and V is more complicated.

* General Relation between $\vec{\mathbf{E}}$ and V

In a region where $\vec{\mathbf{E}}$ is not uniform, the connection between $\vec{\mathbf{E}}$ and V takes on a different form than Eqs. 17–4. In general, it is possible to show that the electric field in a given direction at any point in space is equal to the *rate at which the electric potential decreases over distance in that direction*. For example, the x component of the electric field is given by $E_x = -\Delta V/\Delta x$, where ΔV is the change in potential over the very short distance Δx.

17–3 Equipotential Lines

The electric potential can be represented diagrammatically by drawing **equipotential lines** or, in three dimensions, **equipotential surfaces**. An equipotential surface is one on which all points are at the same potential. That is, the potential difference between any two points on the surface is zero, and no work is required to move a charge from one point to another on an equipotential surface. An *equipotential surface must be perpendicular to the electric field* at any point. If this were not so—that is, if there were a component of $\vec{\mathbf{E}}$ parallel to the surface—it would require work to move the charge along the surface against this component of $\vec{\mathbf{E}}$; and this would contradict the idea that it is an *equi*potential surface.

Equipotentials $\perp \vec{\mathbf{E}}$