(*b*) Inside the shell, the field must also be symmetric. So E must again have the same value at all points on a spherical gaussian surface (A_2 in Fig. 16–39) concentric with the shell. Thus, E can be factored out of the sum and, with $Q_{\text{encl}} = 0$ since the charge inside the surface is zero, we have

$$\sum E_{\perp}\,\Delta A = E \sum \Delta A = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\epsilon_0} = 0.$$

Hence

$$E = 0 \qquad \left[r < r_0\right]$$

inside a uniform spherical shell of charge.

The useful results of Example 16–11 also apply to a uniform *solid* spherical conductor that is charged, since all the charge would lie in a thin layer at the surface (Section 16–9).

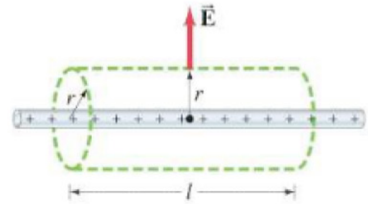


FIGURE 16–40 Calculation of $\vec{\mathbf{E}}$ due to a very long line of charge, Exercise H, where the cylinder shown (dashed) is the gaussian surface.

EXERCISE H A very long, straight wire possesses a uniform charge per unit length, Q/L. Show that the electric field at points near (but outside) the wire, far from the ends, is given by

$$E = \frac{1}{2\pi\epsilon_0 r}\frac{Q}{L}$$

using the cylindrical gaussian surface shown (dashed) in Fig. 16–40. [*Hint:* there is no electric flux through the flat ends of the cylinder.]

EXAMPLE 16–12 ***E* at surface of conductor.** Show that the electric field just outside the surface of any good conductor of arbitrary shape is given by

$$E = \frac{\sigma}{\epsilon_0},$$

where σ is the surface charge density (Q/A) on the conductor at that point.

APPROACH We choose as our gaussian surface a small cylindrical box, very small in height so that one of its circular ends is just above the conductor (Fig. 16–41). The other end is just below the conductor's surface, and the sides are perpendicular to it.

SOLUTION The electric field is zero inside a conductor and is perpendicular to the surface just outside it (Section 16–9), so electric flux passes only through the outside end of our cylindrical box; no flux passes through the short sides or inside end. We choose the area A (of the flat cylinder end above the conductor surface) small enough so that E is essentially uniform over it. Then Gauss's law gives

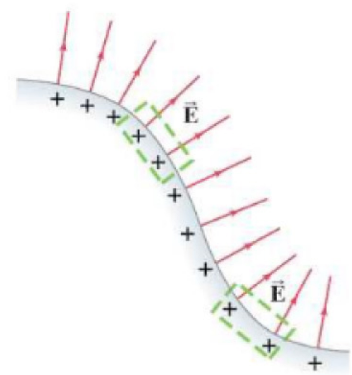
$$\sum E_{\perp}\,\Delta A = EA = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0},$$

so that

$$E = \frac{\sigma}{\epsilon_0}. \qquad \text{[at surface of conductor]}$$

This useful result applies for any shape conductor, including a large, uniformly charged flat sheet: the electric field will be constant and equal to σ/ϵ_0.

FIGURE 16–41 Electric field near the surface of a conductor. Two small cylindrical boxes are shown dashed. Either one can serve as our gaussian surface. Example 16–12.



Electric field at surface of charged conductor

This last Example also gives us the field between the two parallel plates we discussed in Fig. 16–31d. If the plates are large compared to their separation, then the field lines are perpendicular to the plates and, except near the edges, they are parallel to each other. Therefore the electric field (see Fig. 16–42, which shows the same gaussian surface as Fig. 16–41) is also

Oppositely charged parallel plates

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon_0}, \qquad \left[\begin{array}{l}\text{between two closely spaced}\\ \text{oppositely charged parallel plates}\end{array}\right] \quad \textbf{(16–10)}$$

where $Q = \sigma A$ is the charge on one of the plates.

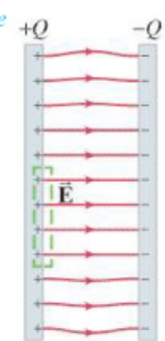


FIGURE 16–42 The electric field between two parallel plates is uniform and equal to $E = \sigma/\epsilon_0$.