Unit for charge: the coulomb

The SI unit of charge is the **coulomb** (C).[†] The precise definition of the coulomb today is in terms of electric current and magnetic field, and will be discussed later (Section 20–6). In SI units, k has the value

$$k = 8.988 \times 10^9\,\mathrm{N \cdot m^2/C^2}$$

or, when we only need two significant figures,

$$k \approx 9.0 \times 10^9\,\mathrm{N \cdot m^2/C^2}.$$

Thus, 1 C is that amount of charge which, if placed on each of two point objects that are 1.0 m apart, will result in each object exerting a force of $(9.0 \times 10^9\,\mathrm{N \cdot m^2/C^2})(1.0\,\mathrm{C})(1.0\,\mathrm{C})/(1.0\,\mathrm{m})^2 = 9.0 \times 10^9\,\mathrm{N}$ on the other. This would be an enormous force, equal to the weight of almost a million tons. We don't normally encounter charges as large as a coulomb.

Charges produced by rubbing ordinary objects (such as a comb or plastic ruler) are typically around a microcoulomb $(1\,\mu\mathrm{C} = 10^{-6}\,\mathrm{C})$ or less. Objects that carry a positive charge have a deficit of electrons, whereas negatively charged objects have an excess of electrons. The charge on one electron has been determined to have a magnitude of about $1.602 \times 10^{-19}\,\mathrm{C}$, and is negative. This is the smallest charge found in nature,[‡] and because it is fundamental, it is given the symbol e and is often referred to as the *elementary charge*:

Charge on electron (the elementary charge)

$$e = 1.602 \times 10^{-19}\,\mathrm{C}.$$

Electric charge is quantized

Note that e is defined as a positive number, so the charge on the electron is $-e$. (The charge on a proton, on the other hand, is $+e$.) Since an object cannot gain or lose a fraction of an electron, the net charge on any object must be an integral multiple of this charge. Electric charge is thus said to be **quantized** (existing only in discrete amounts: $1e$, $2e$, $3e$, etc.). Because e is so small, however, we normally don't notice this discreteness in macroscopic charges ($1\,\mu\mathrm{C}$ requires about 10^{13} electrons), which thus seem continuous.

Coulomb's law and the law of universal gravitation

Coulomb's law looks a lot like the *law of universal gravitation*, $F = G\,m_1 m_2/r^2$, which expresses the gravitational force a mass m_1 exerts on a mass m_2 (Eq. 5–4). Both are inverse square laws $(F \propto 1/r^2)$. Both also have a proportionality to a property of each object—mass for gravity, electric charge for electricity. And both act over a distance (that is, there is no need for contact). A major difference between the two laws is that gravity is always an attractive force, whereas the electric force can be either attractive or repulsive. Electric charge comes in two types, positive and negative; gravitational mass is only positive.

The constant k in Eq. 16–1 is often written in terms of another constant, ϵ_0, called the **permittivity of free space**. It is related to k by $k = 1/4\pi\epsilon_0$. Coulomb's law can then be written

COULOMB'S LAW (in terms of ϵ_0)

$$F = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2}, \qquad \textbf{(16–2)}$$

where

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}.$$

Equation 16–2 looks more complicated than Eq. 16–1, but other fundamental equations we haven't seen yet are simpler in terms of ϵ_0 rather than k. It doesn't matter which form we use since Eqs. 16–1 and 16–2 are equivalent. (The latest precise values of e and ϵ_0 are given inside the front cover.)

Writing units

[Our convention for units, such as $\mathrm{C^2/N \cdot m^2}$ for ϵ_0, means $\mathrm{m^2}$ is in the denominator. That is, $\mathrm{C^2/N \cdot m^2}$ does *not* mean $(\mathrm{C^2/N}) \cdot \mathrm{m^2} = \mathrm{C^2 \cdot m^2/N}$. (Otherwise we would have written it that way.)]

[†] In the once common cgs system of units, k is set equal to 1, and the unit of electric charge is called the *electrostatic unit* (esu) or the statcoulomb. One esu is defined as that charge, on each of two point objects 1 cm apart, that gives rise to a force of 1 dyne.

[‡] According to the standard model of elementary particle physics, subnuclear particles called quarks (Chapter 32) have a smaller charge than that on the electron, equal to $\frac{1}{3}e$ or $\frac{2}{3}e$. Quarks have not been detected directly as isolated objects, and theory indicates that free quarks may not be detectable.