SOLUTION A quantity of heat,

$$Q = mc\,\Delta T = (50.0\,\text{kg})(4186\,\text{J/kg}\cdot\text{C}^\circ)(2.00\,\text{C}^\circ) = 4.186 \times 10^5\,\text{J},$$

flows out of the hot water as it cools down from 24°C to 22°C, and this heat flows into the cold water as it warms from 20°C to 22°C. The total change in entropy, ΔS, will be the sum of the changes in entropy of the hot water, ΔS_{H}, and that of the cold water, ΔS_{C}:

$$\Delta S = \Delta S_{\text{H}} + \Delta S_{\text{C}}.$$

We estimate entropy changes by writing $\Delta S = Q/T_{\text{av}}$, where T_{av} is an "average" temperature for each process, which ought to give a reasonable estimate since the temperature change is small. For the hot water we use an average temperature of 23°C (296 K), and for the cold water an average temperature of 21°C (294 K). Thus

$$\Delta S_{\text{H}} \approx -\frac{4.186 \times 10^5\,\text{J}}{296\,\text{K}} = -1414\,\text{J/K}$$

which is negative because this heat flows out, whereas heat is added to the cold water:

$$\Delta S_{\text{C}} \approx \frac{4.186 \times 10^5\,\text{J}}{294\,\text{K}} = 1424\,\text{J/K}.$$

Note that the entropy of the hot water (S_{H}) decreases since heat flows out of the hot water. But the entropy of the cold water (S_{C}) increases by a greater amount. The total change in entropy is

$$\Delta S = \Delta S_{\text{H}} + \Delta S_{\text{C}} \approx -1414\,\text{J/K} + 1424\,\text{J/K} \approx 10\,\text{J/K}.$$

In Example 15–15, we saw that although the entropy of one part of the system decreased, the entropy of the other part increased by a greater amount; the net change in entropy of the whole system was positive. This result, which we have calculated for a specific case in Example 15–15, has been found to hold in all other cases tested. That is, the total entropy of an isolated system is found to increase in all natural processes. The second law of thermodynamics can be stated in terms of entropy as follows: *The entropy of an isolated system never decreases. It can only stay the same or increase.* Entropy can remain the same only for an idealized (reversible) process. For any real process, the change in entropy ΔS is greater than zero:

Entropy of an isolated system never decreases

$$\Delta S > 0. \tag{15–9}$$

If the system is not isolated, then the change in entropy of the system, ΔS_{s}, plus the change in entropy of the environment, ΔS_{env}, must be greater than or equal to zero:

$$\Delta S = \Delta S_{\text{s}} + \Delta S_{\text{env}} \geq 0. \tag{15–10}$$

Only idealized processes have $\Delta S = 0$. Real processes have $\Delta S > 0$. This, then, is the *general statement of the second law of thermodynamics*:

SECOND LAW OF THERMODYNAMICS (general statement)

the total entropy of any system plus that of its environment increases as a result of any natural process.

Although the entropy of one part of the universe may decrease in any process (see Example 15–15), the entropy of some other part of the universe always increases by a greater amount, so the total entropy always increases.

Now that we finally have a quantitative general statement of the second law of thermodynamics, we can see that it is an unusual law. It differs considerably from other laws of physics, which are typically equalities (such as $F = ma$) or conservation laws (such as for energy and momentum). The second law of thermodynamics introduces a new quantity, the entropy S, but does not tell us it is conserved. Quite the opposite. Entropy is *not* conserved in natural processes; it always increases in time.