Δx can be measured off the graph for this chosen Δt and is found to be 40 m. Thus, the slope of the curve at $t = 10.0\,\text{s}$, which equals the instantaneous velocity at that instant, is $v = \Delta x/\Delta t = 40\,\text{m}/4.0\,\text{s} = 10\,\text{m/s}$.

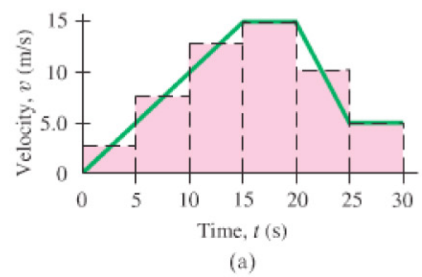
In the region between A and B (Fig. 2–25b) the x vs. t graph is a straight line because the slope (equal to the velocity) is constant. The slope can be measured using the triangle shown for the time interval between $t = 17\,\text{s}$ and $t = 20\,\text{s}$, where the increase in x is 45 m: $\Delta x/\Delta t = 45\,\text{m}/3.0\,\text{s} = 15\,\text{m/s}$.

The slope of an x vs. t graph at any point is $\Delta x/\Delta t$ and thus equals the velocity of the object being described at that moment. Similarly, the slope at any point of a v vs. t graph is $\Delta v/\Delta t$ and so (by Eq. 2–4) equals the acceleration at that moment.

Suppose we were given the x vs. t graph of Fig. 2–25b. We could measure the slopes at a number of points and plot these slopes as a function of time. Since the slope equals the velocity, we could thus reconstruct the v vs. t graph! In other words, given the graph of x vs. t, we can determine the velocity as a function of time using graphical methods, instead of using equations. This technique is particularly useful when the acceleration is not constant, for then Eqs. 2–11 cannot be used.

If, instead, we are given the v vs. t graph, as in Fig. 2–25a, we can determine the position, x, as a function of time using a graphical procedure, which we illustrate by applying it to the v vs. t graph of Fig. 2–25a. We divide the total time interval into subintervals, as shown in Fig. 2–26a, where only six are shown (by dashed vertical lines). In each interval, a *horizontal* dashed line is drawn to indicate the average velocity during that time interval. For example, in the first interval, the velocity increases at a constant rate from zero to 5.0 m/s, so $\overline{v} = 2.5\,\text{m/s}$; and in the fourth interval the velocity is a constant 15 m/s, so $\overline{v} = 15\,\text{m/s}$ (no horizontal dashed line is shown in Fig. 2–26a since it coincides with the curve itself). The displacement (change in position) during any subinterval is $\Delta x = \overline{v}\,\Delta t$. Thus the displacement during each subinterval equals the product of $\overline{v}$ and Δt, which is just the *area of the rectangle* (height × base $= \overline{v} \times \Delta t$), shown shaded in rose, for that interval. The total displacement after 25 s, say, will be the sum of the areas of the first five rectangles.

If the velocity varies a great deal, it may be difficult to estimate $\overline{v}$ from the graph. To reduce this difficulty, we can choose to divide the time interval into many more—but narrower—subintervals of time, making each Δt smaller as shown in Fig. 2–26b. More intervals give a better approximation. Ideally, we could let Δt approach zero; this leads to the techniques of integral calculus, which we don't discuss here. The result, in any case, is that *the total displacement between any two times is equal to the area under the v vs. t graph between these two times.*



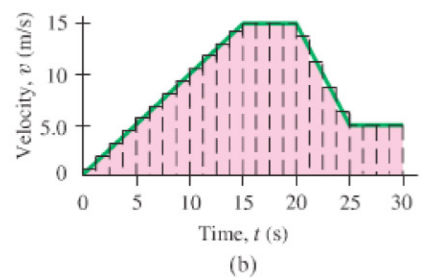


FIGURE 2–26 Determining the displacement from the graph of v vs. t is done by calculating areas.

Displacement = area under v vs. t graph

EXAMPLE 2–16 Displacement using v vs. t graph. A space probe accelerates uniformly from 50 m/s at $t = 0$ to 150 m/s at $t = 10\,\text{s}$. How far did it move between $t = 2.0\,\text{s}$ and $t = 6.0\,\text{s}$?

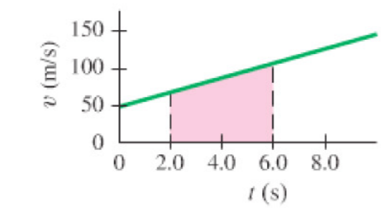
APPROACH A graph of v vs. t can be drawn as shown in Fig. 2–27. We need to calculate the area of the shaded region, which is a trapezoid. The area will be the average of the heights (in units of velocity) times the width (which is 4.0 s).

SOLUTION The acceleration is $a = (150\,\text{m/s} - 50\,\text{m/s})/10\,\text{s} = 10\,\text{m/s}^2$. Using Eq. 2–11a, or Fig. 2–27, at $t = 2.0\,\text{s}$, $v = 70\,\text{m/s}$; and at $t = 6.0\,\text{s}$, $v = 110\,\text{m/s}$. Thus the area, $(\overline{v} \times \Delta t)$, which equals Δx, is

$$\Delta x = \left(\frac{70\,\text{m/s} + 110\,\text{m/s}}{2}\right)(4.0\,\text{s}) = 360\,\text{m}.$$

NOTE For this case of constant acceleration, we could use Eqs. 2–11 and we would get the same result.

FIGURE 2–27 Example 2–16. The shaded area represents the displacement during the time interval $t = 2.0\,\text{s}$ to $t = 6.0\,\text{s}$.



In cases where the acceleration is not constant, the area can be obtained by counting squares on graph paper.