

Fig. 13–26. We assume the molecules are in random motion. Yet there will be a net flow of molecules to the right. To see why this is true, let us consider the small section of tube of length  $\Delta x$  as shown. Molecules from both regions 1 and 2 cross into this central section as a result of their random motion. The more molecules there are in a region, the more will strike a given area or cross a boundary. Since there is a greater concentration of molecules in region 1 than in region 2, more molecules cross into the central section from region 1 than from region 2. There is, then, a net flow of molecules from left to right, from high concentration toward low concentration. The net flow becomes zero only when the concentrations become equal.

You might expect that the greater the difference in concentration, the greater the flow rate. Indeed, the rate of diffusion,  $J$  (number of molecules or moles or kg per second), is directly proportional to the change in concentration per unit distance,  $(C_1 - C_2)/\Delta x$  (which is called the **concentration gradient**), and to the cross-sectional area  $A$  (see Fig. 13–26):

$$J = DA \frac{C_1 - C_2}{\Delta x} \quad (13-10)$$

$D$  is a constant of proportionality called the **diffusion constant**. Equation 13–10 is known as the **diffusion equation**, or **Fick's law**. If the concentrations are given in mol/m<sup>3</sup>, then  $J$  is the number of moles passing a given point per second. If the concentrations are given in kg/m<sup>3</sup>, then  $J$  is the mass movement per second (kg/s). The length  $\Delta x$  is given in meters. The values of  $D$  for a variety of substances are given in Table 13–4.

**EXAMPLE 13–19 ESTIMATE Diffusion of ammonia in air.** To get an idea of the time required for diffusion, estimate how long it might take for ammonia (NH<sub>3</sub>) to be detected 10 cm from a bottle after it is opened, assuming only diffusion is occurring.

**APPROACH** This will be an order-of-magnitude calculation. The rate of diffusion  $J$  can be set equal to the number of molecules  $N$  diffusing across area  $A$  in a time  $t$ :  $J = N/t$ . Then the time  $t = N/J$ , where  $J$  is given by Eq. 13–10. We will have to make some assumptions and rough approximations about concentrations to use Eq. 13–10.

**SOLUTION** Using Eq. 13–10, we get

$$t = \frac{N}{J} = \frac{N \Delta x}{DA \Delta C}$$

The average concentration (midway between bottle and nose) can be approximated by  $\bar{C} \approx N/V$ , where  $V$  is the volume over which the molecules move and is roughly of the order of  $V \approx A \Delta x$ , where  $\Delta x$  is 10 cm = 0.10 m. We substitute  $N = \bar{C}V = \bar{C}A \Delta x$  into the above equation:

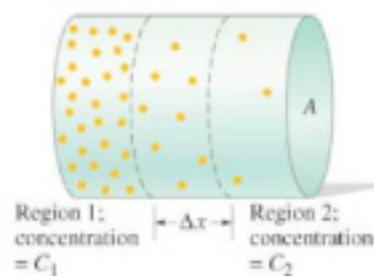
$$t \approx \frac{(\bar{C}A \Delta x)\Delta x}{DA \Delta C} = \frac{\bar{C}}{\Delta C} \frac{(\Delta x)^2}{D}$$

The concentration of ammonia is high near the bottle and low near the detecting nose, so  $\bar{C} \approx \Delta C/2$ , or  $(\bar{C}/\Delta C) \approx \frac{1}{2}$ . Since NH<sub>3</sub> molecules have a size somewhere between H<sub>2</sub> and O<sub>2</sub>, from Table 13–4 we can estimate  $D \approx 4 \times 10^{-5}$  m<sup>2</sup>/s. Then

$$t \approx \frac{1}{2} \frac{(0.10 \text{ m})^2}{(4 \times 10^{-5} \text{ m}^2/\text{s})} \approx 100 \text{ s,}$$

or about a minute or two.

**NOTE** This result seems rather long from experience, suggesting that air currents (convection) are more important than diffusion for transmitting odors.



**FIGURE 13–26** Diffusion occurs from a region of high concentration to one of lower concentration. (Only one type of molecule is shown.)

#### Diffusion equation

#### PHYSICS APPLIED Diffusion time

**TABLE 13–4** Diffusion Constants,  $D$  (20°C, 1 atm)

Diffusing Molecules	Medium	$D$ (m <sup>2</sup> /s)
H <sub>2</sub>	Air	$6.3 \times 10^{-5}$
O <sub>2</sub>	Air	$1.8 \times 10^{-5}$
O <sub>2</sub>	Water	$100 \times 10^{-11}$
Blood hemoglobin	Water	$6.9 \times 10^{-11}$
Glycine (an amino acid)	Water	$95 \times 10^{-11}$
DNA (mass $6 \times 10^6$ u)	Water	$0.13 \times 10^{-11}$