Equation 13–6, $P = \frac{1}{3}N m\overline{v^2}/V$, can be rewritten in a clearer form by multiplying both sides by V and rearranging the right-hand side:

$$PV = \frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right). \tag{13–7}$$

The quantity $\frac{1}{2}m\overline{v^2}$ is the average kinetic energy ($\overline{\text{KE}}$) of the molecules in the gas. If we compare Eq. 13–7 with Eq. 13–4, the ideal gas law $PV = NkT$, we see that the two agree if

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$$\frac{2}{3}\left(\frac{1}{2}m\overline{v^2}\right) = kT,$$

or

$$\overline{\text{KE}} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT. \qquad \text{[ideal gas]} \tag{13–8}$$

This equation tells us that

the average translational kinetic energy of molecules in random motion in an ideal gas is directly proportional to the absolute temperature of the gas.

The higher the temperature, according to kinetic theory, the faster the molecules are moving on the average. This relation is one of the triumphs of the kinetic theory.

EXAMPLE 13–16 **Molecular kinetic energy.** What is the average translational kinetic energy of molecules in an ideal gas at 37°C?

APPROACH We use the absolute temperature in Eq. 13–8.

SOLUTION We change 37°C to 310 K and insert into Eq. 13–8:

$$\overline{\text{KE}} = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23}\,\text{J/K})(310\,\text{K}) = 6.42 \times 10^{-21}\,\text{J}.$$

NOTE A mole of molecules would have a total translational kinetic energy equal to $(6.42 \times 10^{-21}\,\text{J})(6.02 \times 10^{23}) = 3900\,\text{J}$, which equals the kinetic energy of a 1-kg stone traveling faster than 85 m/s.

Equation 13–8 holds not only for gases, but also applies reasonably accurately to liquids and solids. Thus the result of Example 13–16 would apply to molecules within living cells at body temperature (37°C).

Root-mean-square (rms) speed

We can use Eq. 13–8 to calculate how fast molecules are moving on the average. Notice that the average in Eqs. 13–5 through 13–8 is over the *square* of the speed. The square root of $\overline{v^2}$ is called the **root-mean-square** speed, v_{rms} (since we are taking the square *root* of the *mean* of the *square* of the speed):

rms speed of molecules

$$v_{\text{rms}} = \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}}. \tag{13–9}$$

EXAMPLE 13–17 **Speeds of air molecules.** What is the rms speed of air molecules (O_2 and N_2) at room temperature (20°C)?

APPROACH To obtain v_{rms}, we need the masses of O_2 and N_2 molecules and then apply Eq. 13–9 to oxygen and nitrogen separately, since they have different masses.

SOLUTION The masses of one molecule of O_2 (molecular mass = 32 u) and N_2 (molecular mass = 28 u) are (where $1\,\text{u} = 1.66 \times 10^{-27}\,\text{kg}$)

$$m(O_2) = (32)(1.66 \times 10^{-27}\,\text{kg}) = 5.3 \times 10^{-26}\,\text{kg},$$
$$m(N_2) = (28)(1.66 \times 10^{-27}\,\text{kg}) = 4.6 \times 10^{-26}\,\text{kg}.$$

Thus, for oxygen

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{(3)(1.38 \times 10^{-23}\,\text{J/K})(293\,\text{K})}{(5.3 \times 10^{-26}\,\text{kg})}} = 480\,\text{m/s},$$

and for nitrogen the result is $v_{\text{rms}} = 510\,\text{m/s}$. These speeds† are more than 1700 km/h or 1000 mi/h.

†The speed v_{rms} is a magnitude only. The *velocity* of molecules averages to zero: the velocity has direction, and as many molecules move to the right as to the left.