* 2–8 Graphical Analysis of Linear Motion†

Figure 2–9 showed the graph of the velocity of a car versus time for two cases of linear motion: (a) constant velocity, and (b) a particular case in which the magnitude of the velocity varied. It is also useful to graph, or "plot," the position x (or y) as a function of time, as we did in Fig. 2–23a. The time t is considered the independent variable and is measured along the horizontal axis. The position, x, the dependent variable, is measured along the vertical axis.

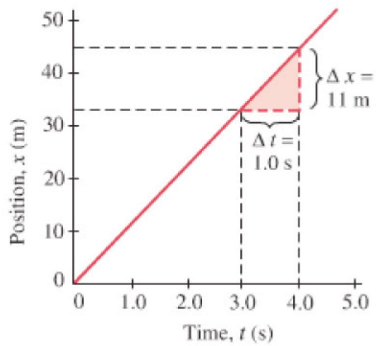

FIGURE 2–24 Graph of position vs. time for an object moving at a uniform velocity of 11 m/s.

Let us make a graph of x vs. t, and make the choice that at $t = 0$, the position is $x_0 = 0$. First we consider a car moving at a constant velocity of 40 km/h, which is equivalent to 11 m/s. Equation 2–11b tells us $x = vt$, and we see that x increases by 11 m every second. Thus, the position increases linearly in time, so the graph of x vs. t is a straight line, as shown in Fig. 2–24. Each point on this straight line tells us the car's position at a particular time. For example, at $t = 3.0\,\text{s}$, the position is 33 m, and at $t = 4.0\,\text{s}$, $x = 44\,\text{m}$, as indicated by the dashed lines. The small (shaded) triangle on the graph indicates the **slope** of the straight line, which is defined as the change in the dependent variable (Δx) divided by the corresponding change in the independent variable (Δt):

$$\text{slope} = \frac{\Delta x}{\Delta t}.$$

Velocity = slope of x vs. t graph

We see, using the definition of average velocity (Eq. 2–2), that the *slope of the x vs. t graph is equal to the velocity*. And, as can be seen from the small triangle on the graph, $\Delta x/\Delta t = (11\,\text{m})/(1.0\,\text{s}) = 11\,\text{m/s}$, which is the given velocity.

The slope of the x vs. t graph is everywhere the same if the velocity is constant, as in Fig. 2–24. But if the velocity changes, as in Fig. 2–25a, the slope of the x vs. t graph also varies. Consider, for example, a car that (1) accelerates uniformly from rest to 15 m/s in 15 s, after which (2) it remains at a constant velocity of 15 m/s for the next 5.0 s; (3) during the following 5.0 s, the car slows down uniformly to 5.0 m/s, and then (4) remains at this constant velocity. This velocity as a function of time is shown in the graph of Fig. 2–25a. To construct the x vs. t graph, we can use Eq. 2–11b ($x = x_0 + v_0 t + \frac{1}{2}at^2$) with constant acceleration for the interval $t = 0$ to $t = 15\,\text{s}$ and for $t = 20\,\text{s}$ to $t = 25\,\text{s}$; for the constant velocity period $t = 15\,\text{s}$ to $t = 20\,\text{s}$, and after $t = 25\,\text{s}$, we set $a = 0$. The result is the x vs. t graph of Fig. 2–25b.

Slope of a curve

From the origin to point A, the x vs. t graph (Fig. 2–25b) is not a straight line, but is curved. The **slope** of a curve at any point is defined as the *slope of the tangent to the curve at that point*. (The *tangent* is a straight line drawn so it touches the curve only at that one point, but does not pass across or through the curve.) For example, the tangent to the x vs. t curve at the time $t = 10.0\,\text{s}$ is drawn on the graph of Fig. 2–25b. A triangle is drawn with Δt chosen to be 4.0 s;

†Some Sections of this book, such as this one, may be considered *optional* at the discretion of the instructor. See the Preface for more details.

FIGURE 2–25 (a) Velocity vs. time and (b) displacement vs. time for an object with variable velocity. (See text.)