Next we solve Eq. 2–6 for t, obtaining

$$t = \frac{v - v_0}{a},$$

and substituting this into the previous equation we have

$$x = x_0 + \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right) = x_0 + \frac{v^2 - v_0^2}{2a}.$$

We solve this for v^2 and obtain

$$v^2 = v_0^2 + 2a(x - x_0), \qquad \text{[constant acceleration]} \quad \textbf{(2–10)}$$

v related to a and x (a = constant)

which is the useful equation we sought.

We now have four equations relating position, velocity, acceleration, and time, when the acceleration a is constant. We collect these kinematic equations here in one place for future reference (the tan background screen emphasizes their usefulness):

$$v = v_0 + at \qquad [a = \text{constant}] \quad \textbf{(2–11a)}$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad [a = \text{constant}] \quad \textbf{(2–11b)}$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad [a = \text{constant}] \quad \textbf{(2–11c)}$$

$$\bar{v} = \frac{v + v_0}{2}. \qquad [a = \text{constant}] \quad \textbf{(2–11d)}$$

Kinematic equations for constant acceleration (we'll use them a lot)

These useful equations are not valid unless a is a constant. In many cases we can set $x_0 = 0$, and this simplifies the above equations a bit. Note that x represents position, not distance, that $x - x_0$ is the displacement, and that t is the elapsed time.

EXAMPLE 2–6 **Runway design.** You are designing an airport for small planes. One kind of airplane that might use this airfield must reach a speed before takeoff of at least 27.8 m/s (100 km/h), and can accelerate at 2.00 m/s². (a) If the runway is 150 m long, can this airplane reach the required speed for take off? (b) If not, what minimum length must the runway have?

PHYSICS APPLIED

Airport design

APPROACH The plane's acceleration is given as constant $(a = 2.00\ \text{m/s}^2)$, so we can use the kinematic equations for constant acceleration. In (a), we are given that the plane can travel a distance of 150 m. The plane starts from rest, so $v_0 = 0$ and we take $x_0 = 0$. We want to find its velocity, to determine if it will be at least 27.8 m/s. We want to find v when we are given:

Known	Wanted
$x_0 = 0$	v
$v_0 = 0$	
$x = 150\ \text{m}$	
$a = 2.00\ \text{m/s}^2$	

SOLUTION (a) Of the above four equations, Eq. 2–11c will give us v when we know v_0, a, x, and x_0:

PROBLEM SOLVING

Equations 2–11 are valid only when the acceleration is constant, which we assume in this Example

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$= 0 + 2(2.0\ \text{m/s}^2)(150\ \text{m}) = 600\ \text{m}^2/\text{s}^2$$

$$v = \sqrt{600\ \text{m}^2/\text{s}^2} = 24.5\ \text{m/s}.$$

This runway length is *not* sufficient.

(b) Now we want to find the minimum length of runway, $x - x_0$, given $v = 27.8\ \text{m/s}$ and $a = 2.00\ \text{m/s}^2$. So we again use Eq. 2–11c, but rewritten as

$$(x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{(27.8\ \text{m/s})^2 - 0}{2(2.0\ \text{m/s}^2)} = 193\ \text{m}.$$

A 200-m runway is more appropriate for this plane.