2–5 Motion at Constant Acceleration

Let a = constant

Many practical situations occur in which the acceleration is constant or nearly constant. We now examine this situation when the magnitude of the acceleration is constant and the motion is in a straight line. In this case, the instantaneous and average accelerations are equal.

We now use our definitions of velocity and acceleration to derive a set of extremely useful equations that relate x, v, a, and t when a is constant, allowing us to determine any one of these variables if we know the others.

x (at $t = 0$) $= x_0$
v (at $t = 0$) $= v_0$
t = elapsed time

To simplify our notation, let us take the initial time in any discussion to be zero, and we call it t_0: $t_1 = t_0 = 0$. (This is effectively starting a stopwatch at t_0.) We can then let $t_2 = t$ be the elapsed time. The initial position (x_1) and the initial velocity (v_1) of an object will now be represented by x_0 and v_0, since they represent x and v at $t = 0$. At time t the position and velocity will be called x and v (rather than x_2 and v_2). The average velocity during the time interval $t - t_0$ will be (Eq. 2–2)

$$\bar{v} = \frac{x - x_0}{t - t_0} = \frac{x - x_0}{t}$$

since we chose $t_0 = 0$. The acceleration, assumed constant in time, is (Eq. 2–4)

$$a = \frac{v - v_0}{t}.$$

A common problem is to determine the velocity of an object after any elapsed time t, when we are given the object's constant acceleration. We can solve such problems by solving for v in the last equation to obtain:

v related to a and t (a = constant, t = elapsed time)

$$v = v_0 + at. \qquad \text{[constant acceleration]} \quad \textbf{(2–6)}$$

For example, it may be known that the acceleration of a particular motorcycle is $4.0\,\text{m/s}^2$, and we wish to determine how fast it will be going after an elapsed time $t = 6.0\,\text{s}$ when it starts from rest ($v_0 = 0$ at $t_0 = 0$). At $t = 6.0\,\text{s}$, the velocity will be $v = at = (4.0\,\text{m/s}^2)(6.0\,\text{s}) = 24\,\text{m/s}$.

Next, let us see how to calculate the position of an object after a time t when it is undergoing constant acceleration. The definition of average velocity (Eq. 2–2) is $\bar{v} = (x - x_0)/t$, which we can rewrite as

$$x = x_0 + \bar{v}t. \qquad \textbf{(2–7)}$$

Because the velocity increases at a uniform rate, the average velocity, $\bar{v}$, will be midway between the initial and final velocities:

CAUTION
Average velocity, but only if a = constant

$$\bar{v} = \frac{v_0 + v}{2}. \qquad \text{[constant acceleration]} \quad \textbf{(2–8)}$$

(Careful: Eq. 2–8 is not necessarily valid if the acceleration is not constant.) We combine the last two Equations with Eq. 2–6 and find

$$\begin{aligned} x &= x_0 + \bar{v}t = x_0 + \left(\frac{v_0 + v}{2}\right)t \\ &= x_0 + \left(\frac{v_0 + v_0 + at}{2}\right)t \end{aligned}$$

or

x related to a and t (a = constant)

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2. \qquad \text{[constant acceleration]} \quad \textbf{(2–9)}$$

Equations 2–6, 2–8, and 2–9 are three of the four most useful equations for motion at constant acceleration. We now derive the fourth equation, which is useful in situations where the time t is not known. We begin with Eq. 2–7 and substitute in Eq. 2–8:

$$x = x_0 + \bar{v}t = x_0 + \left(\frac{v + v_0}{2}\right)t.$$