EXAMPLE 8–4 Angular and linear velocities and accelerations. A carousel is initially at rest. At $t = 0$ it is given a constant angular acceleration $\alpha = 0.060\ \text{rad/s}^2$, which increases its angular velocity for 8.0 s. At $t = 8.0\ \text{s}$, determine the following quantities: (*a*) the angular velocity of the carousel; (*b*) the linear velocity of a child (Fig. 8–7a) located 2.5 m from the center, point P in Fig. 8–7b; (*c*) the tangential (linear) acceleration of that child; (*d*) the centripetal acceleration of the child; and (*e*) the total linear acceleration of the child.

APPROACH The angular acceleration α is constant, so we can use Eq. 8–3a to solve for ω after a time $t = 8.0\ \text{s}$. With this ω and the given α, we determine the other quantities using the relations we just developed, Eqs. 8–4, 8–5, and 8–6.

SOLUTION (*a*) Equation 8–3a tells us

$$\overline{\alpha} = \frac{\omega_2 - \omega_1}{\Delta t}.$$

We are given $\Delta t = 8.0\ \text{s}$, $\overline{\alpha} = 0.060\ \text{rad/s}^2$, and $\omega_1 = 0$. Solving for ω_2, we get

$$\begin{aligned}\omega_2 &= \omega_1 + \overline{\alpha}\,\Delta t\\ &= 0 + (0.060\ \text{rad/s}^2)(8.0\ \text{s}) = 0.48\ \text{rad/s}.\end{aligned}$$

During the 8.0-s interval, the carousel has accelerated from $\omega_1 = 0$ (rest) to $\omega_2 = 0.48\ \text{rad/s}$.

(*b*) The linear velocity of the child with $r = 2.5\ \text{m}$ at time $t = 8.0\ \text{s}$ is found using Eq. 8–4:

$$v = r\omega = (2.5\ \text{m})(0.48\ \text{rad/s}) = 1.2\ \text{m/s}.$$

Note that the "rad" has been dropped here because it is dimensionless (and only a reminder)—it is a ratio of two distances, Eq. 8–1b.

(*c*) The child's tangential acceleration is given by Eq. 8–5:

$$a_{\tan} = r\alpha = (2.5\ \text{m})(0.060\ \text{rad/s}^2) = 0.15\ \text{m/s}^2,$$

and it is the same throughout the 8.0-s acceleration interval.

(*d*) The child's centripetal acceleration at $t = 8.0\ \text{s}$ is given by Eq. 8–6:

$$a_{\text{R}} = \frac{v^2}{r} = \frac{(1.2\ \text{m/s})^2}{(2.5\ \text{m})} = 0.58\ \text{m/s}^2.$$

(*e*) The two components of linear acceleration calculated in parts (*c*) and (*d*) are perpendicular to each other. Thus the total linear acceleration at $t = 8.0\ \text{s}$ has magnitude

$$\begin{aligned}a &= \sqrt{a_{\tan}^2 + a_{\text{R}}^2}\\ &= \sqrt{(0.15\ \text{m/s}^2)^2 + (0.58\ \text{m/s}^2)^2} = 0.60\ \text{m/s}^2.\end{aligned}$$

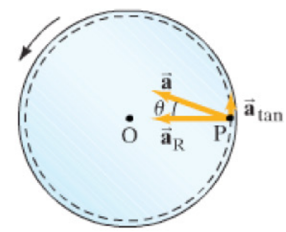
Its direction (Fig. 8–7b) is

$$\theta = \tan^{-1}\left(\frac{a_{\tan}}{a_{\text{R}}}\right) = \tan^{-1}\left(\frac{0.15\ \text{m/s}^2}{0.58\ \text{m/s}^2}\right) = 0.25\ \text{rad},$$

so $\theta \approx 15°$.

NOTE The linear acceleration is mostly centripetal, keeping the child moving in a circle with the carousel. The tangential component that speeds up the motion is smaller.

(a)



(b)

FIGURE 8–7 Example 8–4. The total acceleration vector $\vec{\mathbf{a}} = \vec{\mathbf{a}}_{\tan} + \vec{\mathbf{a}}_{\text{R}}$, at $t = 8.0\ \text{s}$.