conceptual example 8-3 Is the lion faster than the horse? On a rotating carousel or merry-go-round, one child sits on a horse near the outer edge and another child sits on a lion halfway out from the center. (a) Which child has the greater linear velocity? (b) Which child has the greater angular velocity?

RESPONSE (a) The *linear* velocity is the distance traveled divided by the time interval. In one rotation the child on the outer edge travels a longer distance than the child near the center, but the time interval is the same for both. Thus the child at the outer edge, on the horse, has the greater linear velocity.

(b) The angular velocity is the angle of rotation divided by the time interval. In one rotation both children rotate through the same angle ($360^{\circ} = 2\pi$ radians). The two children have the same angular velocity.

If the angular velocity of a rotating object changes, the object as a whole—and each point in it—has an angular acceleration. Each point also has a linear acceleration whose direction is tangent to that point's circular path. We use Eq. 8–4 $(v = r\omega)$ to show that the angular acceleration α is related to the tangential linear acceleration a_{tan} of a point in the rotating object by

$$a_{\rm tan} = \frac{\Delta v}{\Delta t} = r \frac{\Delta \omega}{\Delta t}$$

or

$$a_{tan} = r\alpha.$$
 (8-5)

In this equation, r is the radius of the circle in which the particle is moving, and the subscript "tan" in a_{tan} stands for "tangential."

The total linear acceleration of a point is the vector sum of two components:

$$\vec{\mathbf{a}} = \vec{\mathbf{a}}_{tan} + \vec{\mathbf{a}}_{R}$$

where the radial[†] component, \vec{a}_R , is the radial or "centripetal" acceleration and its direction is toward the center of the point's circular path; see Fig. 8–6. We saw in Chapter 5 (Eq. 5–1) that $a_R = v^2/r$, and we can rewrite this in terms of ω using Eq. 8–4:

Centripetal (or radial) acceleration

Tangential acceleration

$$a_{\rm R} = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \omega^2 r.$$
 (8-6)

Thus the centripetal acceleration is greater the farther you are from the axis of rotation: the children farthest out on a carousel feel the greatest acceleration. Equations 8–4, 8–5, and 8–6 relate the angular quantities describing the rotation of an object to the linear quantities for each point of the object. Table 8–1 summarizes these relationships.

a tan P

FIGURE 8-6 On a rotating wheel

whose angular speed is increasing, a point P has both tangential and radial

(centripetal) components of linear acceleration. (See also Chapter 5.)

| TABLE 8-1 Linear and Rotational Quantities | | | |
|--|--------------|------------|-----------------|
| Linear | Туре | Rotational | Relation |
| х | displacement | θ | $x = r\theta$ |
| v | velocity | ω | $v = r\omega$ |
| a_{tan} | acceleration | α | $a_{\tan} = ra$ |

^{*&}quot;Radial" means along the radius-that is, toward or away from the center or axis.