Angular acceleration (denoted by α, the Greek lowercase letter alpha), in analogy to linear acceleration, is defined as the change in angular velocity divided by the time required to make this change. The **average angular acceleration** is defined as

$$\overline{\alpha} = \frac{\omega_2 - \omega_1}{\Delta t} = \frac{\Delta\omega}{\Delta t}, \tag{8-3a}$$

where ω_1 is the angular velocity initially, and ω_2 is the angular velocity after a time interval Δt. **Instantaneous angular acceleration** is defined in the usual way as the limit of this ratio as Δt approaches zero:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}. \tag{8-3b}$$

Angular acceleration

Since ω is the same for all points of a rotating object, Eq. 8–3 tells us that α also will be the same for all points. Thus, ω and α are properties of the rotating object as a whole. With ω measured in radians per second and t in seconds, α will be expressed as radians per second squared (rad/s^2).

Each point or particle of a rotating rigid object has, at any moment, a linear velocity v and a linear acceleration a. We can relate the linear quantities at each point, v and a, to the angular quantities of the rotating object, ω and α. Consider a point P located a distance r from the axis of rotation, as in Fig. 8–4. If the object rotates with angular velocity ω, any point will have a linear velocity whose direction is tangent to its circular path. The magnitude of that point's linear velocity is $v = \Delta l/\Delta t$. From Eq. 8–1b, a change in rotation angle $\Delta\theta$ (in radians) is related to the linear distance traveled by $\Delta l = r\,\Delta\theta$. Hence

$$v = \frac{\Delta l}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$$

or

$$v = r\omega. \tag{8-4}$$

Linear and angular velocity related

Thus, although ω is the same for every point in the rotating object at any instant, the linear velocity v is greater for points farther from the axis (Fig. 8–5). Note that Eq. 8–4 is valid both instantaneously and on the average.

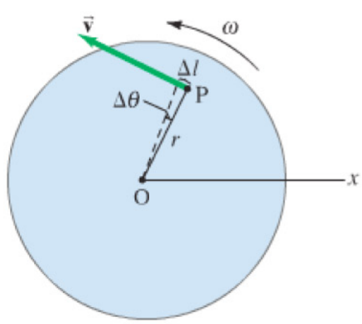

FIGURE 8–4 A point P on a rotating wheel has a linear velocity $\vec{\mathbf{v}}$ at any moment.

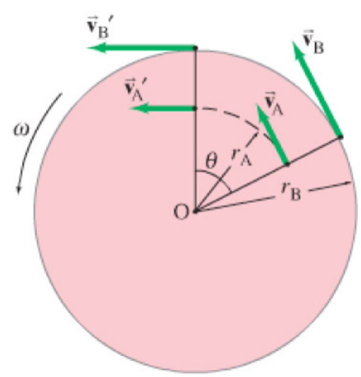

FIGURE 8–5 A wheel rotating uniformly counterclockwise. Two points on the wheel, at distances r_A and r_B from the center, have the same angular velocity ω because they travel through the same angle θ in the same time interval. But the two points have different linear velocities because they travel different distances in the same time interval. Since $r_B > r_A$, then $v_B > v_A$ $(v = r\omega)$.