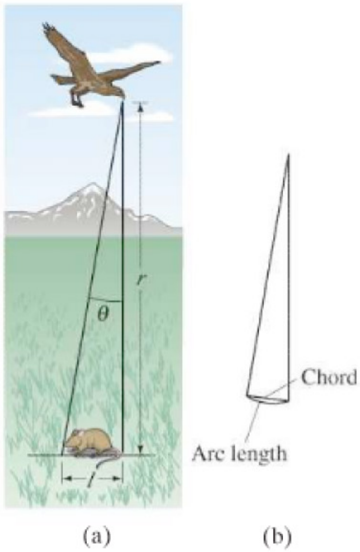


FIGURE 8–2 (a) Example 8–2. (b) For small angles, arc length and the chord length (straight line) are nearly equal.

EXAMPLE 8–2 **Birds of prey—in radians.** A particular bird's eye can just distinguish objects that subtend an angle no smaller than about 3×10^{-4} rad. (*a*) How many degrees is this? (*b*) How small an object can the bird just distinguish when flying at a height of 100 m (Fig. 8–2a)?

APPROACH For (*a*) we use the relation $360° = 2\pi$ rad. For (*b*) we use Eq. 8–1b, $l = r\theta$, to find the arc length.

SOLUTION (*a*) We convert 3×10^{-4} rad to degrees:

$$(3 \times 10^{-4}\,\text{rad})\left(\frac{360°}{2\pi\,\text{rad}}\right) = 0.017°.$$

(*b*) We use Eq. 8–1b, $l = r\theta$. For small angles, the arc length l and the chord length are approximately[†] the same (Fig. 8–2b). Since $r = 100$ m and $\theta = 3 \times 10^{-4}$ rad, we find

$$l = (100\,\text{m})(3 \times 10^{-4}\,\text{rad}) = 3 \times 10^{-2}\,\text{m} = 3\,\text{cm}.$$

A bird can distinguish a small mouse (about 3 cm long) from a height of 100 m. That is good eyesight.

NOTE Had the angle been given in degrees, we would first have had to convert it to radians to make this calculation. Equation 8–1 is valid *only* if the angle is specified in radians. Degrees (or revolutions) won't work.

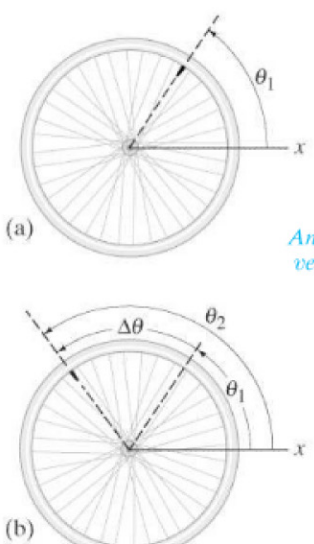
To describe rotational motion, we make use of angular quantities, such as angular velocity and angular acceleration. These are defined in analogy to the corresponding quantities in linear motion, and are chosen to describe the rotating object as a whole, so they are the same for each point in the rotating object. Each point in a rotating object may also have translational velocity and acceleration, but they have different values for different points in the object.

Angular displacement (rad)

When an object, such as the bicycle wheel in Fig. 8–3, rotates from some initial position, specified by θ_1, to some final position, θ_2, its *angular displacement* is

$$\Delta\theta = \theta_2 - \theta_1.$$

FIGURE 8–3 A wheel rotates from (a) initial position θ_1 to (b) final position θ_2. The angular displacement is $\Delta\theta = \theta_2 - \theta_1$.



The *angular velocity* (denoted by ω, the Greek lowercase letter omega) is defined in analogy with linear (translational) velocity that was discussed in Chapter 2. Instead of linear displacement, we use the angular displacement. Thus the **average angular velocity** is defined as

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}, \tag{8–2a}$$

where $\Delta\theta$ is the angle through which the object has rotated in the time interval Δt. We define the **instantaneous angular velocity** as the very small angle $\Delta\theta$, through which the object turns in the very short time interval Δt:

Angular velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}. \tag{8–2b}$$

Angular velocity is generally specified in radians per second (rad/s). Note that *all points in a rigid object rotate with the same angular velocity*, since every position in the object moves through the same angle in the same time interval.

An object such as the wheel in Fig. 8–3 can rotate about a fixed axis either clockwise or counterclockwise. The direction can be specified with a + or − sign, just as we did in Chapter 2 for linear motion along the $+x$ or $-x$ axis. The usual convention is to choose the angular displacement $\Delta\theta$ and angular velocity ω as positive when the wheel rotates counterclockwise. If the rotation is clockwise, then θ would decrease, so $\Delta\theta$ and ω would be negative.[‡]

[†] Even for an angle as large as 15°, the error in making this estimate is only 1%, but for larger angles the error increases rapidly.

[‡] The vector nature of angular velocity and other angular quantities is discussed in Section 8–9 (optional).