8–1 Angular Quantities

We saw in Chapter 7 (Section 7–8) that the motion of a rigid object can be analyzed as the translational motion of the object's center of mass, plus rotational motion *about* its center of mass. We have already discussed translational motion in detail, so now we focus on purely rotational motion. By *purely rotational motion*, we mean that all points in the object move in circles, such as the point P in the rotating wheel of Fig. 8–1, and that the centers of these circles all lie on a line called the **axis of rotation**. In Fig. 8–1 the axis of rotation is perpendicular to the page and passes through point O.

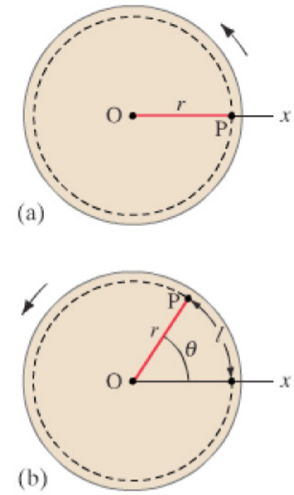

FIGURE 8–1 Looking at a wheel that is rotating counterclockwise about an axis through the wheel's center at O (axis perpendicular to the page). Each point, such as point P, moves in a circular path; l is the distance P travels as the wheel rotates through the angle θ.

Every point in an object rotating about a fixed axis moves in a circle (shown dashed in Fig. 8–1 for point P) whose center is on the axis and whose radius is r, the distance of that point from the axis of rotation. A straight line drawn from the axis to any point sweeps out the same angle θ in the same time.

To indicate the angular position of a rotating object, or how far it has rotated, we specify the angle θ of some particular line in the object (red in Fig. 8–1) with respect to a reference line, such as the x axis in Fig. 8–1. A point in the object, such as P in Fig. 8–1, moves through an angle θ when it travels the distance l measured along the circumference of its circular path. Angles are commonly measured in degrees, but the mathematics of circular motion is much simpler if we use the *radian* for angular measure. One **radian** (abbreviated rad) is defined as the angle subtended by an arc whose length is equal to the radius. For example, in Fig. 8–1b, point P is a distance r from the axis of rotation, and it has moved a distance l along the arc of a circle. The arc length l is said to "subtend" the angle θ. If $l = r$, then θ is exactly equal to 1 rad. In radians, any angle θ is given by

$$\theta = \frac{l}{r}, \tag{8–1a}$$

θ in radians

where r is the radius of the circle, and l is the arc length subtended by the angle specified in radians. If $l = r$, then $\theta = 1$ rad.

1 rad: arc length = radius

The radian is dimensionless since it is the ratio of two lengths. Nonetheless when giving an angle in radians, we always mention rad to remind us it is not degrees. It is often useful to rewrite Eq. 8–1a in terms of arc length l:

$$l = r\theta. \tag{8–1b}$$

Radians can be related to degrees in the following way. In a complete circle there are 360°, which must correspond to an arc length equal to the circumference of the circle, $l = 2\pi r$. Thus $\theta = l/r = 2\pi r/r = 2\pi\,\text{rad}$ in a complete circle, so

$$360° = 2\pi\,\text{rad}.$$

Conversion, degrees to rad

One radian is therefore $360°/2\pi \approx 360°/6.28 \approx 57.3°$. An object that makes one complete revolution (rev) has rotated through 360°, or 2π radians:

1 rad ≈ 57.3°

$$1\,\text{rev} = 360° = 2\pi\,\text{rad}.$$

EXAMPLE 8–1 **Bike wheel.** A bike wheel rotates 4.50 revolutions. How many radians has it rotated?

APPROACH All we need is a straightforward conversion of units using

$$1\,\text{revolution} = 360° = 2\pi\,\text{rad} = 6.28\,\text{rad}.$$

SOLUTION

$$4.50\,\text{revolutions} = (4.50\,\text{rev})\left(2\pi\frac{\text{rad}}{\text{rev}}\right) = 9.00\pi\,\text{rad} = 28.3\,\text{rad}.$$