If forces are acting on the particles, then the particles may be accelerating. In a short time interval Δt , each particle's velocity will change by an amount

$$\Delta v_{\rm A} = a_{\rm A} \, \Delta t, \qquad \Delta v_{\rm B} = a_{\rm B} \, \Delta t, \qquad \Delta v_{\rm C} = a_{\rm C} \, \Delta t.$$

If we now use the same reasoning as we did to derive Eq. 7-10, we obtain

$$Ma_{\rm CM} = m_{\rm A} a_{\rm A} + m_{\rm B} a_{\rm B} + m_{\rm C} a_{\rm C}.$$

According to Newton's second law, $m_A a_A = F_A$, $m_B a_B = F_B$, and $m_C a_C = F_C$, where F_A , F_B , and F_C are the net forces on the three particles, respectively. Thus we get for the system as a whole $Ma_{CM} = F_A + F_B + F_C$, or

Newton's second law for a system of particles or an extended object

$$Ma_{\rm CM} = F_{\rm net}. ag{7-11}$$

That is, the sum of all the forces acting on the system is equal to the total mass of the system times the acceleration of its center of mass. This is Newton's second law for a system of particles, and it also applies to an extended object (which can be thought of as a collection of particles). Thus we conclude that the center of mass of a system of particles (or of an extended object) with total mass M moves like a single particle of mass M acted on by the same net external force. That is, the system moves as if all its mass were concentrated at the center of mass and all the external forces acted at that point. We can thus treat the translational motion of any object or system of objects as the motion of a particle (see Figs. 7-21 and 7-22). This result simplifies our analysis of the motion of complex systems and extended objects. Although the motion of various parts of the system may be complicated, we may often be satisfied with knowing the motion of the center of mass. This result also allows us to solve certain types of problems very easily, as illustrated by the following Example.

CONCEPTUAL EXAMPLE 7-14 A two-stage rocket. A rocket is shot into the air as shown in Fig. 7-29. At the moment the rocket reaches its highest point, a horizontal distance d from its starting point, a prearranged explosion separates it into two parts of equal mass. Part I is stopped in midair by the explosion, and it falls vertically to Earth. Where does part II land? Assume $\vec{\mathbf{g}} = \text{constant}$.

RESPONSE After the rocket is fired, the path of the CM of the system continues to follow the parabolic trajectory of a projectile acted on by only a constant gravitational force. The CM will thus land at a point 2d from the starting point. Since the masses of I and II are equal, the CM must be midway between them at any time. Therefore, part II lands a distance 3d from the starting point.

NOTE If part I had been given a kick up or down, instead of merely falling, the solution would have been more complicated.

EXERCISE H A woman stands up in a rowboat and walks from one end of the boat to the other. How does the boat move, as seen from the shore?

FIGURE 7-29 Example 7-14.

