For our 1.70-m-tall person, this is $(1.70\,\text{m})(14.9/100) = 0.25\,\text{m}$ from the hip joint. Next, we calculate the distance, y_{CM}, of the CM above the floor:

$$y_{\text{CM}} = \frac{(3.4)(1.8) + (9.6)(18.2) + (21.5)(28.5)}{21.5 + 9.6 + 3.4} = 23.0\ \text{units},$$

or $(1.70\,\text{m})(23.0/100) = 0.39\,\text{m}$. Thus, the CM is located 39 cm above the floor and 25 cm to the right of the hip joint.

NOTE The CM actually lies *outside* the body in (*b*).

FIGURE 7–28 A high jumper's CM may actually pass beneath the bar.

PHYSICS APPLIED
High jumping

Knowing the CM of the body when it is in various positions is of great use in studying body mechanics. One simple example from athletics is shown in Fig. 7–28. If high jumpers can get into the position shown, their CM can pass below the bar which their bodies go over, meaning that for a particular takeoff speed, they can clear a higher bar. This is indeed what they try to do.

* 7–10 Center of Mass and Translational Motion

As mentioned in Section 7–8, a major reason for the importance of the concept of center of mass is that the motion of the CM for a system of particles (or an extended object) is directly related to the net force acting on the system as a whole. We now show this, taking the simple case of one-dimensional motion (x direction) and only three particles, but the extension to more objects and to three dimensions follows the same lines.

Suppose the three particles lie on the x axis and have masses m_A, m_B, m_C, and positions x_A, x_B, x_C. From Eq. 7–9a for the center of mass, we can write

$$Mx_{\text{CM}} = m_A x_A + m_B x_B + m_C x_C, \qquad \textbf{(i)}$$

where $M = m_A + m_B + m_C$ is the total mass of the system. If these particles are in motion (say, along the x axis with velocities v_A, v_B, and v_C, respectively), then in a short time interval Δt they each will have traveled a distance

$$\Delta x_A = x'_A - x_A = v_A\,\Delta t$$
$$\Delta x_B = x'_B - x_B = v_B\,\Delta t$$
$$\Delta x_C = x'_C - x_C = v_C\,\Delta t,$$

where x'_A, x'_B, and x'_C represent their new positions after time interval Δt. The position of the new CM is given by

$$Mx'_{\text{CM}} = m_A x'_A + m_B x'_B + m_C x'_C. \qquad \textbf{(ii)}$$

If we subtract from this equation (ii) the previous CM equation (i), we get

$$M\,\Delta x_{\text{CM}} = m_A\,\Delta x_A + m_B\,\Delta x_B + m_C\,\Delta x_C.$$

During time interval Δt, the center of mass will have moved a distance

$$\Delta x_{\text{CM}} = x'_{\text{CM}} - x_{\text{CM}} = v_{\text{CM}}\,\Delta t,$$

where v_{CM} is the velocity of the center of mass. We now substitute the relations for all the Δx's into the equation just before the last one:

$$Mv_{\text{CM}}\,\Delta t = m_A v_A\,\Delta t + m_B v_B\,\Delta t + m_C v_C\,\Delta t.$$

We cancel Δt and get

$$Mv_{\text{CM}} = m_A v_A + m_B v_B + m_C v_C. \qquad \textbf{(7–10)}$$

Total momentum, and velocity of CM

Since $m_A v_A + m_B v_B + m_C v_C$ is the sum of the momenta of the particles of the system, it represents the *total momentum* of the system. Thus we see from Eq. 7–10 that *the total (linear) momentum of a system of particles is equal to the product of the total mass M and the velocity of the center of mass of the system.* Or, *the linear momentum of an extended object is the product of the object's mass and the velocity of its CM.*