The second equation yields [recall that $\sin(-\theta) = -\sin\theta$]:

$$v'_B = -v'_A \frac{\sin(45°)}{\sin(-45°)} = -v'_A\left(\frac{\sin 45°}{-\sin 45°}\right) = v'_A.$$

So they do have equal speeds as we guessed at first. The x component equation gives [recall that $\cos(-\theta) = \cos\theta$]:

$$v_A = v'_A\cos(45°) + v'_B\cos(45°) = 2v'_A\cos(45°),$$

so

$$v'_A = v'_B = \frac{v_A}{2\cos(45°)} = \frac{3.0\ \text{m/s}}{2(0.707)} = 2.1\ \text{m/s}.$$

NOTE When we have two independent equations, we can solve for, at most, two unknowns.

EXERCISE F Make a calculation to see if kinetic energy was conserved in the collision of Example 7–11.

If we know that a collision is elastic, we can also apply conservation of kinetic energy and obtain a third equation in addition to Eqs. 7–8a and b:

$$\text{KE}_A + \text{KE}_B = \text{KE}'_A + \text{KE}'_B$$

or, for the collision shown in Fig. 7–20,

$$\tfrac{1}{2}m_A v_A^2 = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2. \qquad \text{[elastic collision]} \quad \textbf{(7–8c)}$$

KE conserved

If the collision is elastic, we have three independent equations and can solve for three unknowns. If we are given m_A, m_B, v_A (and v_B, if it is not zero), we cannot, for example, predict the final variables, v'_A, v'_B, θ'_A, and θ'_B, because there are four of them. However, if we measure one of these variables, say θ'_A, then the other three variables (v'_A, v'_B, and θ'_B) are uniquely determined, and we can determine them using Eqs. 7–8a, b, c.

A note of caution: Eq. 7–7 does *not* apply for two-dimensional collisions. It works only when a collision occurs along a line.

CAUTION
Equation 7–7 applies only in 1-D

PROBLEM SOLVING Momentum Conservation and Collisions

1. Choose your **system**. If the situation is complex, think about how you might break it up into separate parts when one or more conservation laws apply.
2. Consider whether a significant **net external force** acts on your chosen system; if it does, be sure the time interval Δt is so short that the effect on momentum is neglible. That is, the forces that act between the interacting objects must be the significant ones if momentum conservation is to be used. [Note: If this is valid for a portion of the problem, you can use momentum conservation only for that portion.]
3. Draw a **diagram** of the initial situation, just before the interaction (collision, explosion) takes place, and represent the momentum of each object with an arrow and a label. Do the same for the final situation, just after the interaction.
4. Choose a **coordinate system** and "+" and "−" directions. (For a head-on collision, you will need only an x axis.) It is often convenient to choose the $+x$ axis in the direction of one object's initial velocity.
5. Apply the **momentum conservation** equation(s):

   total initial momentum = total final momentum.

   You have one equation for each component (x, y, z): only one equation for a head-on collision. [Don't forget that it is the *total* momentum of the system that is conserved, not the momenta of individual objects.]
6. If the collision is elastic, you can also write down a **conservation of kinetic energy** equation:

   total initial KE = total final KE.

   [Alternately, you could use Eq. 7–7: $v_A - v_B = v'_B - v'_A$, if the collision is one dimensional (head-on).]
7. Solve for the **unknown(s)**.
8. **Check** your work, check the units, and ask yourself whether the results are reasonable.