In part (1), Fig. 7–17a, we assume the collision time is very short, so the projectile comes to rest in the block before the block has moved significantly from its position directly below its support. Thus there is effectively no net external force, and we can apply conservation of momentum to this completely inelastic collision. In part (2), Fig. 7–17b, the pendulum begins to move, subject to a net external force (gravity, tending to pull it back to the vertical position); so for part (2), we cannot use conservation of momentum. But we can use conservation of mechanical energy because gravity is a conservative force (Chapter 6). The kinetic energy immediately after the collision is changed entirely to gravitational potential energy when the pendulum reaches its maximum height, h.

SOLUTION In part (1) momentum is conserved:

$$\text{total } p \text{ before} = \text{total } p \text{ after}$$

$$mv = (m + M)v', \qquad \textbf{(i)}$$

where v' is the speed of the block and embedded projectile just after the collision, before they have moved significantly.

In part (2), mechanical energy is conserved. We choose $y = 0$ when the pendulum hangs vertically, and then $y = h$ when the pendulum–projectile system reaches its maximum height. Thus we write

$$(\text{KE} + \text{PE}) \text{ just after collision} = (\text{KE} + \text{PE}) \text{ at pendulum's maximum height}$$

or

$$\tfrac{1}{2}(m + M)v'^2 \;+\; 0 = 0 \;+\; (m + M)gh. \qquad \textbf{(ii)}$$

We solve for v':

$$v' = \sqrt{2gh}.$$

Inserting this result for v' into Eq. (i) above, and solving for v, gives

$$v = \frac{m + M}{m}v' = \frac{m + M}{m}\sqrt{2gh},$$

which is our final result.

NOTE The separation of the process into two parts was crucial. Such an analysis is a powerful problem-solving tool. But how do you decide how to make such a division? Think about the conservation laws. They are your *tools*. Start a problem by asking yourself whether the conservation laws apply in the given situation. Here, we determined that momentum is conserved only during the brief collision, which we called part (1). But in part (1), because the collision is inelastic, the conservation of mechanical energy is not valid. Then in part (2), conservation of mechanical energy is valid, but not conservation of momentum.

Note, however, that if there had been significant motion of the pendulum during the deceleration of the projectile in the block, then there *would* have been an external force (gravity) during the collision, so conservation of momentum would not have been valid in part (1).

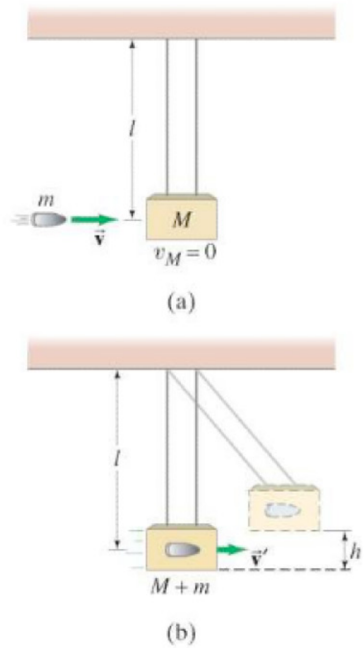

FIGURE 7–17 Ballistic pendulum. Example 7–10.

PROBLEM SOLVING
Use the conservation laws to analyze a problem

* 7–7 Collisions in Two or Three Dimensions

Conservation of momentum and energy can also be applied to collisions in two or three dimensions, where the vector nature of momentum is especially important. One common type of non-head-on collision is that in which a moving object (called the "projectile") strikes a second object initially at rest (the "target"). This is the common situation in games such as billiards and pool, and for experiments in atomic and nuclear physics (the projectiles, from radioactive decay or a high-energy accelerator, strike a stationary target nucleus; Fig. 7–18).

FIGURE 7–18 A recent color-enhanced version of a cloud-chamber photograph made in the early days (1920s) of nuclear physics. Green lines are paths of helium nuclei (He) coming from the left. One He, highlighted in yellow, strikes a proton of the hydrogen gas in the chamber, and both scatter at an angle; the scattered proton's path is shown in red.