7–2 Conservation of Momentum

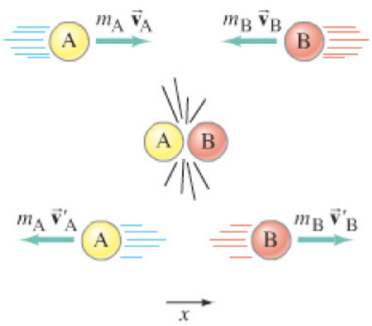


FIGURE 7–3 Momentum is conserved in a collision of two balls, labelled A and B.

The concept of momentum is particularly important because, under certain circumstances, momentum is a conserved quantity. Consider, for example, the head-on collision of two billiard balls, as shown in Fig. 7–3. We assume the net external force on this system of two balls is zero—that is, the only significant forces during the collision are the forces that each ball exerts on the other. Although the momentum of each of the two balls changes as a result of the collision, the *sum* of their momenta is found to be the same before as after the collision. If $m_A \vec{\mathbf{v}}_A$ is the momentum of ball A and $m_B \vec{\mathbf{v}}_B$ the momentum of ball B, both measured just before the collision, then the total momentum of the two balls before the collision is the vector sum $m_A \vec{\mathbf{v}}_A + m_B \vec{\mathbf{v}}_B$. Immediately after the collision, the balls each have a different velocity and momentum, which we designate by a "prime" on the velocity: $m_A \vec{\mathbf{v}}'_A$ and $m_B \vec{\mathbf{v}}'_B$. The total momentum after the collision is the vector sum $m_A \vec{\mathbf{v}}'_A + m_B \vec{\mathbf{v}}'_B$. No matter what the velocities and masses are, experiments show that the total momentum before the collision is the same as afterward, whether the collision is head-on or not, as long as no net external force acts:

CONSERVATION OF MOMENTUM (for two objects colliding)

$$\text{momentum before} = \text{momentum after}$$

$$m_A \vec{\mathbf{v}}_A + m_B \vec{\mathbf{v}}_B = m_A \vec{\mathbf{v}}'_A + m_B \vec{\mathbf{v}}'_B. \qquad \textbf{(7–3)}$$

That is, the total vector momentum of the system of two colliding balls is conserved: it stays constant.

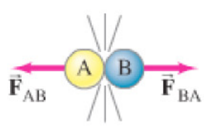
Momentum conservation related to Newton's laws

Although the law of conservation of momentum was discovered experimentally, it is closely connected to Newton's laws of motion and they can be shown to be equivalent. We will do a derivation for the head–on collision illustrated in Fig. 7–3. We assume the force F that one ball exerts on the other during the collision is constant over the brief time interval of the collision Δt. We use Newton's second law as expressed in Eq. 7–2, and rewrite it by multiplying both sides by Δt:

$$\Delta \vec{\mathbf{p}} = \vec{\mathbf{F}}\,\Delta t. \qquad \textbf{(7–4)}$$

We apply this to ball B alone, noting that the force $\vec{\mathbf{F}}_{BA}$ on ball B exerted by ball A during the collision is to the right ($+x$ direction—see Fig. 7–4):

FIGURE 7–4 Forces on the balls during the collision of Fig. 7–3.



$$\Delta \vec{\mathbf{p}}_B = \vec{\mathbf{F}}_{BA}\,\Delta t$$

$$m_B \vec{\mathbf{v}}'_B - m_B \vec{\mathbf{v}}_B = \vec{\mathbf{F}}_{BA}\,\Delta t.$$

By Newton's third law, the force $\vec{\mathbf{F}}_{AB}$ on ball A due to ball B is $\vec{\mathbf{F}}_{AB} = -\vec{\mathbf{F}}_{BA}$ and acts to the left. Then applying Newton's second law in the same way to ball A yields

$$\Delta \vec{\mathbf{p}}_A = \vec{\mathbf{F}}_{AB}\,\Delta t$$

or

$$m_A \vec{\mathbf{v}}'_A - m_A \vec{\mathbf{v}}_A = \vec{\mathbf{F}}_{AB}\,\Delta t$$
$$= -\vec{\mathbf{F}}_{BA}\,\Delta t.$$

We combine these two $\Delta \vec{\mathbf{p}}$ equations (their right sides differ only by a minus sign):

$$m_A \vec{\mathbf{v}}'_A - m_A \vec{\mathbf{v}}_A = -(m_B \vec{\mathbf{v}}'_B - m_B \vec{\mathbf{v}}_B)$$

or

$$m_A \vec{\mathbf{v}}_A + m_B \vec{\mathbf{v}}_B = m_A \vec{\mathbf{v}}'_A + m_B \vec{\mathbf{v}}'_B$$

which is Eq. 7–3, the conservation of momentum.

The above derivation can be extended to include any number of interacting objects. To show this, we let $\vec{\mathbf{p}}$ in Eq. 7–2 represent the total momentum of a system—that is, the vector sum of the momenta of all objects in the system. (For our two-object system above, $\vec{\mathbf{p}} = m_A \vec{\mathbf{v}}_A + m_B \vec{\mathbf{v}}_B$.) If the net force $\Sigma \vec{\mathbf{F}}$ on the system is zero [as it was above for our two-object system, $\vec{\mathbf{F}} + (-\vec{\mathbf{F}}) = 0$,] then