7–1 Momentum and Its Relation to Force

The **linear momentum** (or "momentum" for short) of an object is defined as the product of its mass and its velocity. Momentum (plural is *momenta*) is represented by the symbol $\vec{\mathbf{p}}$. If we let m represent the mass of an object and $\vec{\mathbf{v}}$ represent its velocity, then its momentum $\vec{\mathbf{p}}$ is defined as

Linear momentum defined

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}. \tag{7–1}$$

Velocity is a vector, so momentum too is a vector. The direction of the momentum is the direction of the velocity, and the magnitude of the momentum is $p = mv$. Because velocity depends on the reference frame, so does momentum; thus the reference frame must be specified. The unit of momentum is that of mass × velocity, which in SI units is kg·m/s. There is no special name for this unit.

Units of momentum

Everyday usage of the term *momentum* is in accord with the definition above. According to Eq. 7–1, a fast-moving car has more momentum than a slow-moving car of the same mass; a heavy truck has more momentum than a small car moving with the same speed. The more momentum an object has, the harder it is to stop it, and the greater effect it will have if it is brought to rest by striking another object. A football player is more likely to be stunned if tackled by a heavy opponent running at top speed than by a lighter or slower-moving tackler. A heavy, fast-moving truck can do more damage than a slow-moving motorcycle.

EXERCISE A Can a small sports car ever have the same momentum as a large sport-utility vehicle with three times the sports car's mass? Explain.

A force is required to change the momentum of an object, whether it is to increase the momentum, to decrease it, or to change its direction. Newton originally stated his second law in terms of momentum (although he called the product mv the "quantity of motion"). Newton's statement of the **second law of motion**, translated into modern language, is as follows:

NEWTON'S SECOND LAW

The rate of change of momentum of an object is equal to the net force applied to it.

We can write this as an equation,

NEWTON'S SECOND LAW

$$\Sigma\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}, \tag{7–2}$$

CAUTION
The change in the momentum vector is in the direction of the net force

where $\Sigma\vec{\mathbf{F}}$ is the net force applied to the object (the vector sum of all forces acting on it) and $\Delta\vec{\mathbf{p}}$ is the resulting momentum change that occurs during the time interval† Δt.

We can readily derive the familiar form of the second law, $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$, from Eq. 7–2 for the case of constant mass. If $\vec{\mathbf{v}}_1$ is the initial velocity of an object and $\vec{\mathbf{v}}_2$ is its velocity after a time interval Δt has elapsed, then

$$\Sigma\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t} = \frac{m\vec{\mathbf{v}}_2 - m\vec{\mathbf{v}}_1}{\Delta t} = \frac{m(\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1)}{\Delta t}$$
$$= m\frac{\Delta\vec{\mathbf{v}}}{\Delta t}.$$

By definition, $\vec{\mathbf{a}} = \Delta\vec{\mathbf{v}}/\Delta t$, so

Newton's second law for constant mass

$$\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}. \qquad \text{[constant mass]}$$

Newton's statement, Eq. 7–2, is more general than the more familiar version because it includes the situation in which the mass may change. A change in mass occurs in certain circumstances, such as for rockets which lose mass as they burn fuel, and also in the theory of relativity (Chapter 26).

†Normally we think of Δt as being a small time interval. If it is not small, then Eq. 7–2 is valid if $\Sigma\vec{\mathbf{F}}$ is constant during that time interval, or if $\Sigma\vec{\mathbf{F}}$ is the average net force during that time interval.