Equation 6–13 can be applied to any object moving without friction under the action of gravity. For example, Fig. 6–19 shows a roller-coaster car starting from rest at the top of a hill, and coasting without friction to the bottom and up the hill on the other side.† Initially, the car has only potential energy. As it coasts down the hill, it loses potential energy and gains in kinetic energy, but the sum of the two remains constant. At the bottom of the hill it has its maximum kinetic energy; as it climbs up the other side, the kinetic energy changes back to potential energy. When the car comes to rest again, all of its energy will be potential energy. Given that the potential energy is proportional to the vertical height, energy conservation tells us that (in the absence of friction) the car comes to rest at a height equal to its original height. If the two hills are the same height, the car will just barely reach the top of the second hill when it stops. If the second hill is lower than the first, not all of the car's kinetic energy will be transformed to potential energy and the car can continue over the top and down the other side. If instead the second hill is higher, the car will only reach a height on it equal to its original height on the first hill. This is true (in the absence of friction) no matter how steep the hill is, since potential energy depends only on the vertical height (Eq. 6–6).

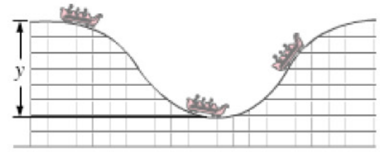

FIGURE 6–19 A roller-coaster car moving without friction illustrates the conservation of mechanical energy.

Grav. PE depends on vertical height, not path length (Eq. 6–6)

EXAMPLE 6–9 Roller-coaster speed using energy conservation. Assuming the height of the hill in Fig. 6–19 is 40 m, and the roller-coaster car starts from rest at the top, calculate (*a*) the speed of the roller-coaster car at the bottom of the hill, and (*b*) at what height it will have half this speed. Take $y = 0$ at the bottom of the hill.

APPROACH We choose point 1 to be where the car starts from rest $(v_1 = 0)$ at the top of the hill $(y_1 = 40\text{ m})$. Point 2 is the bottom of the hill, which we choose as our reference level, so $y_2 = 0$. We use conservation of mechanical energy.

SOLUTION (*a*) We use Eq. 6–13 with $v_1 = 0$ and $y_2 = 0$. Then

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2$$
$$mgy_1 = \tfrac{1}{2}mv_2^2.$$

The m's cancel out and, setting $y_1 = 40\text{ m}$, we find

$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.8\text{ m/s}^2)(40\text{ m})} = 28\text{ m/s}.$$

(*b*) We again use conservation of energy,

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2,$$

but now $v_2 = 14\text{ m/s}$ (half of 28 m/s) and y_2 is unknown. We cancel the m's, set $v_1 = 0$, and solve for y_2:

$$y_2 = y_1 - \frac{v_2^2}{2g} = 30\text{ m}.$$

That is, the car has a speed of 14 m/s when it is 30 *vertical* meters above the lowest point, both when descending the left-hand hill and when ascending the right-hand hill.

NOTE The mathematics of this Example is almost the same as that in Example 6–8. But there is an important difference between them. Example 6–8 could have been solved using force, acceleration, and the kinematic equations (Eqs. 2–11). But here, where the motion is not vertical, that approach would have been too complicated, whereas energy conservation readily gives us the answer.

† The forces on the car are gravity, the normal force exerted by the track, and friction (here, assumed zero). The normal force acts perpendicular to the track, and so is always perpendicular to the motion and does no work. Thus $W_{NC} = 0$ in Eq. 6–10 (so mechanical energy is conserved) and we can use Eq. 6–13 with the potential energy being only gravitational potential energy. We will see how to deal with friction, for which $W_{NC} \neq 0$, in Section 6–9.