In the next Section we shall see the great usefulness of the conservation of mechanical energy principle in a variety of situations, and how it is often easier to use than the kinematic equations or Newton's laws. After that we will discuss how other forms of energy can be included in the general conservation of energy law that includes energy associated with nonconservative forces.

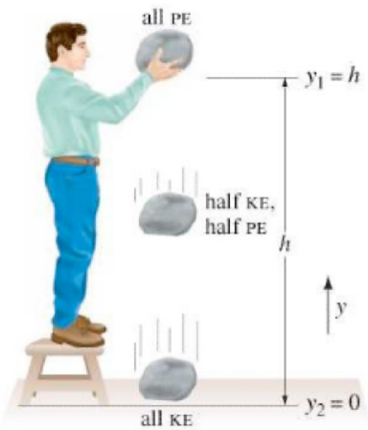


FIGURE 6–17 As it falls, the rock's potential energy changes to kinetic energy.

6–7 Problem Solving Using Conservation of Mechanical Energy

A simple example of the conservation of mechanical energy (neglecting air resistance) is a rock allowed to fall under gravity from a height h above the ground, as shown in Fig. 6–17. If the rock starts from rest, all of the initial energy is potential energy. As the rock falls, the potential energy decreases (because y decreases), but the rock's kinetic energy increases to compensate, so that the sum of the two remains constant. At any point along the path, the total mechanical energy is given by

$$E = \text{KE} + \text{PE} = \tfrac{1}{2}mv^2 + mgy$$

where y is the rock's height above the ground at a given instant and v is its speed at that point. If we let the subscript 1 represent the rock at one point along its path (for example, the initial point), and the subscript 2 represent it at some other point, then we can write

total mechanical energy at point 1 = total mechanical energy at point 2

or (see also Eq. 6–12a)

Conservation of mechanical energy when only gravity acts

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2. \qquad \text{[grav. PE only]} \quad \mathbf{(6–13)}$$

Just before the rock hits the ground, where we chose $y = 0$, all of the initial potential energy will have been transformed into kinetic energy.

EXAMPLE 6–8 **Falling rock.** If the original height of the rock in Fig. 6–17 is $y_1 = h = 3.0\,\text{m}$, calculate the rock's speed when it has fallen to 1.0 m above the ground.

APPROACH One approach is to use the kinematic equations of Chapter 2. Let us instead apply the principle of conservation of mechanical energy, Eq. 6–13, assuming that only gravity acts on the rock. We choose the ground as our reference level ($y = 0$).

SOLUTION At the moment of release (point 1) the rock's position is $y_1 = 3.0\,\text{m}$ and it is at rest: $v_1 = 0$. We want to find v_2 when the rock is at position $y_2 = 1.0\,\text{m}$. Equation 6–13 gives

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2.$$

The m's cancel out; setting $v_1 = 0$ and solving for v_2^2 we find

$$v_2^2 = 2g(y_1 - y_2)$$
$$= 2(9.8\,\text{m/s}^2)[(3.0\,\text{m}) - (1.0\,\text{m})] = 39.2\,\text{m}^2/\text{s}^2,$$

and

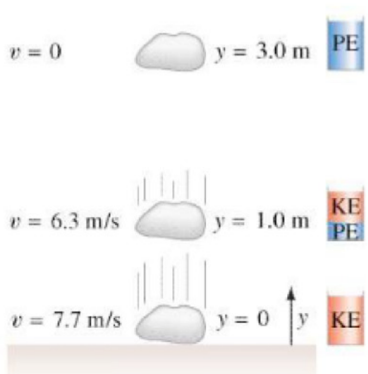
$$v_2 = \sqrt{39.2}\,\text{m/s} = 6.3\,\text{m/s}.$$

The rock's speed 1.0 m above the ground is 6.3 m/s downward.

NOTE The velocity at point 2 is independent of the rock's mass.

EXERCISE D Solve Example 6–8 by using the work-energy principle applied to the rock, without the concept of potential energy. Show all equations you use, starting with Eq. 6–4.

FIGURE 6–18 Energy buckets (for Example 6–8). Kinetic energy is red and potential energy is blue. The total (KE + PE) is the same for the three points shown. The speed at $y = 0$, just before the rock hits the ground, is $\sqrt{2(9.8\,\text{m/s}^2)(3.0\,\text{m})} = 7.7\,\text{m/s}$.



A simple way to visualize energy conservation is with an "energy bucket" as shown in Fig. 6–18. At each point in the fall of the rock, for example, the amount of kinetic energy and potential energy are shown as if they were two differently colored materials in the bucket. The total amount of material in the bucket (= total mechanical energy) remains constant.