

know what the guess is.”⁴ Inaccuracies caused by guessing too low for one number are often canceled out by other guesses that are too high. You will find that with practice your guesstimates get better and better. Estimation problems can be fun to work as you freely drop digits, venture reasonable approximations for unknown numbers, make simplifying assumptions, and turn the question around into something you can answer in your head.

EXAMPLE 1.5 Breaths in a Lifetime

Estimate the number of breaths taken during an average life span.

Solution We shall start by guessing that the typical life span is about 70 years. The only other estimate we must make in this example is the average number of breaths that a person takes in 1 min. This number varies, depending on whether the person is exercising, sleeping, angry, serene, and so forth. To the nearest order of magnitude, we shall choose 10 breaths per minute as our estimate of the average. (This is certainly closer to the true value than 1 breath per minute or 100 breaths per minute.) The number of minutes in a year is

approximately

$$1 \text{ yr} \times 400 \frac{\text{days}}{\text{yr}} \times 25 \frac{\text{h}}{\text{day}} \times 60 \frac{\text{min}}{\text{h}} = 6 \times 10^5 \text{ min}$$

Notice how much simpler it is to multiply 400×25 than it is to work with the more accurate 365×24 . These approximate values for the number of days in a year and the number of hours in a day are close enough for our purposes. Thus, in 70 years there will be $(70 \text{ yr})(6 \times 10^5 \text{ min/yr}) = 4 \times 10^7$ min. At a rate of 10 breaths/min, an individual would take

$$4 \times 10^8 \text{ breaths in a lifetime.}$$

EXAMPLE 1.6 It's a Long Way to San Jose

Estimate the number of steps a person would take walking from New York to Los Angeles.

Solution Without looking up the distance between these two cities, you might remember from a geography class that they are about 3 000 mi apart. The next approximation we must make is the length of one step. Of course, this length depends on the person doing the walking, but we can estimate that each step covers about 2 ft. With our estimated step size, we can determine the number of steps in 1 mi. Because this is a rough calculation, we round 5 280 ft/mi to 5 000 ft/mi. (What percentage error does this introduce?) This conversion factor gives us

$$\frac{5\,000 \text{ ft/mi}}{2 \text{ ft/step}} = 2\,500 \text{ steps/mi}$$

Now we switch to scientific notation so that we can do the calculation mentally:

$$(3 \times 10^3 \text{ mi})(2.5 \times 10^3 \text{ steps/mi}) = 7.5 \times 10^6 \text{ steps} \\ \sim 10^7 \text{ steps}$$

So if we intend to walk across the United States, it will take us on the order of ten million steps. This estimate is almost certainly too small because we have not accounted for curving roads and going up and down hills and mountains. Nonetheless, it is probably within an order of magnitude of the correct answer.

EXAMPLE 1.7 How Much Gas Do We Use?

Estimate the number of gallons of gasoline used each year by all the cars in the United States.

Solution There are about 270 million people in the United States, and so we estimate that the number of cars in the country is 100 million (guessing that there are between two and three people per car). We also estimate that the aver-

age distance each car travels per year is 10 000 mi. If we assume a gasoline consumption of 20 mi/gal or 0.05 gal/mi, then each car uses about 500 gal/yr. Multiplying this by the total number of cars in the United States gives an estimated

$$\text{total consumption of } 5 \times 10^{10} \text{ gal} \sim 10^{11} \text{ gal.}$$

⁴ E. Taylor and J. A. Wheeler, *Spacetime Physics*, San Francisco, W. H. Freeman & Company, Publishers, 1966, p. 60.