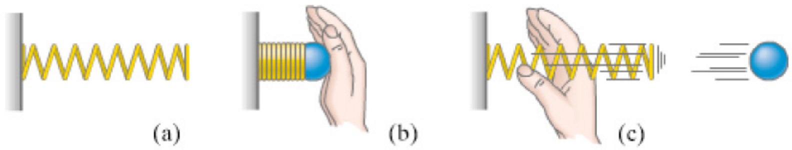

FIGURE 6–13 (a) A spring can store energy (elastic PE) when compressed as in (b) and can do work when released (c).

PE of elastic spring

We now consider another type of potential energy, that associated with elastic materials. This includes a great variety of practical applications. Consider the simple coil spring shown in Fig. 6–13. The spring has potential energy when compressed (or stretched), for when it is released, it can do work on a ball as shown. To hold a spring either stretched or compressed an amount x from its natural (unstretched) length requires the hand to exert a force on the spring, F_P, that is directly proportional to x. That is,

$$F_P = kx,$$

where k is a constant, called the *spring stiffness constant*, and is a measure of the stiffness of the particular spring. The stretched or compressed spring exerts a force F_S in the opposite direction on the hand, as shown in Fig. 6–14:

$$F_S = -kx. \qquad \textbf{(6–8)}$$

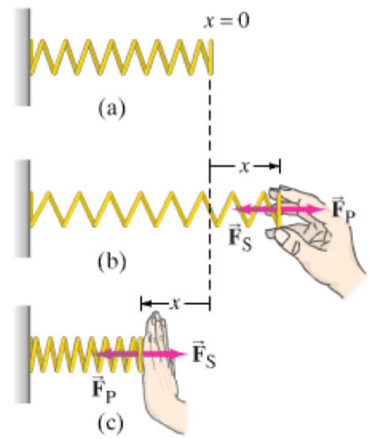

FIGURE 6–14 (a) Spring in natural (unstretched) position. (b) Spring is stretched by a person exerting a force $\vec{F}_P$ to the right (positive direction). The spring pulls back with a force $\vec{F}_S$, where $F_S = -kx$. (c) Person compresses the spring $(x < 0)$ by exerting a force $\vec{F}_P$ to the left; the spring pushes back with a force $F_S = -kx$, where $F_S > 0$ because $x < 0$.

This force is sometimes called a "restoring force" because the spring exerts its force in the direction opposite the displacement (hence the minus sign), acting to return it to its natural length. Equation 6–8 is known as the **spring equation** and also as **Hooke's law**, and is accurate for springs as long as x is not too great.

To calculate the potential energy of a stretched spring, let us calculate the work required to stretch it (Fig. 6–14b). We might expect to use Eq. 6–1 for the work done on it, $W = Fx$, where x is the amount it is stretched from its natural length. But this would be incorrect since the force $F_P\,(= kx)$ is not constant but varies over this distance, becoming greater the more the spring is stretched, as shown graphically in Fig. 6–15. So let us use the average force, $\overline{F}$. Since F_P varies linearly—from zero at the unstretched position to kx when stretched to x—the average force is $\overline{F} = \frac{1}{2}[0 + kx] = \frac{1}{2}kx$, where x here is the final amount stretched (shown as x_f in Fig. 6–15 for clarity). The work done is then

$$W = \overline{F}x = \left(\tfrac{1}{2}kx\right)(x) = \tfrac{1}{2}kx^2.$$

Hence the **elastic potential energy** is proportional to the square of the amount stretched:

Elastic PE

$$\text{elastic PE} = \tfrac{1}{2}kx^2. \qquad \textbf{(6–9)}$$

If a spring is *compressed* a distance x from its natural length, the average force is again $\overline{F} = \frac{1}{2}kx$, and again the potential energy is given by Eq. 6–9. Thus x can be either the amount compressed or amount stretched from the spring's natural length.[†] Note that for a spring, we choose the reference point for zero PE at the spring's natural position.

[†] We can also obtain Eq. 6–9 using Section 6–2. The work done, and hence ΔPE, equals the area under the F vs. x graph of Fig. 6–15. This area is a triangle (colored in Fig. 6–15) of altitude kx and base x, and hence of area (for a triangle) equal to $\frac{1}{2}(kx)(x) = \frac{1}{2}kx^2$.

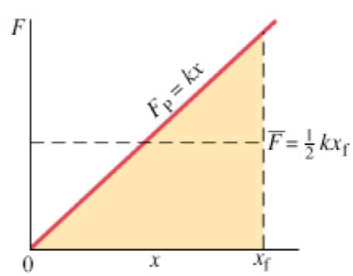

FIGURE 6–15 As a spring is stretched (or compressed), the force needed increases linearly as x increases: graph of $F = kx$ vs. x from $x = 0$ to $x = x_f$.