Quick Quiz 1.1

True or False: Dimensional analysis can give you the numerical value of constants of proportionality that may appear in an algebraic expression.

EXAMPLE 1.2 Analysis of an Equation

Show that the expression $v = at$ is dimensionally correct, where v represents speed, a acceleration, and t a time interval.

Solution For the speed term, we have from Table 1.6

$$[v] = \frac{\text{L}}{\text{T}}$$

The same table gives us L/T^2 for the dimensions of acceleration, and so the dimensions of at are

$$[at] = \left(\frac{\text{L}}{\text{T}^{\not{2}}}\right)(\not{\text{T}}) = \frac{\text{L}}{\text{T}}$$

Therefore, the expression is dimensionally correct. (If the expression were given as $v = at^2$, it would be dimensionally *incorrect*. Try it and see!)

EXAMPLE 1.3 Analysis of a Power Law

Suppose we are told that the acceleration a of a particle moving with uniform speed v in a circle of radius r is proportional to some power of r, say r^n, and some power of v, say v^m. How can we determine the values of n and m?

Solution Let us take a to be

$$a = kr^n v^m$$

where k is a dimensionless constant of proportionality. Knowing the dimensions of a, r, and v, we see that the dimensional equation must be

$$\text{L}/\text{T}^2 = \text{L}^n(\text{L}/\text{T})^m = \text{L}^{n+m}/\text{T}^m$$

This dimensional equation is balanced under the conditions

$$n + m = 1 \qquad \text{and} \qquad m = 2$$

Therefore $n = -1$, and we can write the acceleration expression as

$$a = kr^{-1}v^2 = k\frac{v^2}{r}$$

When we discuss uniform circular motion later, we shall see that $k = 1$ if a consistent set of units is used. The constant k would not equal 1 if, for example, v were in km/h and you wanted a in m/s^2.

1.5 CONVERSION OF UNITS

Sometimes it is necessary to convert units from one system to another. Conversion factors between the SI units and conventional units of length are as follows:

$$1 \text{ mi} = 1\,609 \text{ m} = 1.609 \text{ km} \qquad 1 \text{ ft} = 0.304\,8 \text{ m} = 30.48 \text{ cm}$$

$$1 \text{ m} = 39.37 \text{ in.} = 3.281 \text{ ft} \qquad 1 \text{ in.} \equiv 0.025\,4 \text{ m} = 2.54 \text{ cm (exactly)}$$

A more complete list of conversion factors can be found in Appendix A.

Units can be treated as algebraic quantities that can cancel each other. For example, suppose we wish to convert 15.0 in. to centimeters. Because 1 in. is defined as exactly 2.54 cm, we find that

$$15.0 \text{ in.} = (15.0 \not{\text{in.}})(2.54 \text{ cm}/\not{\text{in.}}) = 38.1 \text{ cm}$$

This works because multiplying by $\left(\frac{2.54 \text{ cm}}{1 \text{ in.}}\right)$ is the same as multiplying by 1, because the numerator and denominator describe identical things.

QuickLab

Estimate the weight (in pounds) of two large bottles of soda pop. Note that 1 L of water has a mass of about 1 kg. Use the fact that an object weighing 2.2 lb has a mass of 1 kg. Find some bathroom scales and check your estimate.