1–7 Order of Magnitude: Rapid Estimating

We are sometimes interested only in an approximate value for a quantity. This might be because an accurate calculation would take more time than it is worth or would require additional data that are not available. In other cases, we may want to make a rough estimate in order to check an accurate calculation made on a calculator, to make sure that no blunders were made when the numbers were entered.

PROBLEM SOLVING
How to make a rough estimate

A rough estimate is made by rounding off all numbers to one significant figure and its power of 10, and after the calculation is made, again only one significant figure is kept. Such an estimate is called an **order-of-magnitude estimate** and can be accurate within a factor of 10, and often better. In fact, the phrase "order of magnitude" is sometimes used to refer simply to the power of 10.

To give you some idea of how useful and powerful rough estimates can be, let us do a few "worked-out Examples."

PHYSICS APPLIED
Estimating the volume (or mass) of a lake; see also Fig. 1–10

EXAMPLE 1–6 ESTIMATE Volume of a lake. Estimate how much water there is in a particular lake, Fig. 1–10a, which is roughly circular, about 1 km across, and you guess it has an average depth of about 10 m.

APPROACH No lake is a perfect circle, nor can lakes be expected to have a perfectly flat bottom. We are only estimating here. To estimate the volume, we can use a simple model of the lake as a cylinder: we multiply the average depth of the lake times its roughly circular surface area, as if the lake were a cylinder (Fig. 1–10b).

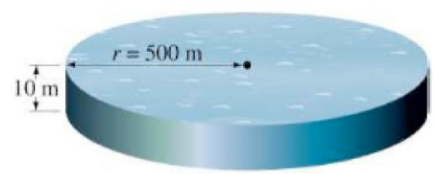
SOLUTION The volume V of a cylinder is the product of its height h times the area of its base: $V = h\pi r^2$, where r is the radius of the circular base.† The radius r is $\frac{1}{2}\,\mathrm{km} = 500\,\mathrm{m}$, so the volume is approximately

$$V = h\pi r^2 \approx (10\,\mathrm{m}) \times (3) \times \left(5 \times 10^2\,\mathrm{m}\right)^2 \approx 8 \times 10^6\,\mathrm{m}^3 \approx 10^7\,\mathrm{m}^3,$$

where π was rounded off to 3. So the volume is on the order of $10^7\,\mathrm{m}^3$, ten million cubic meters. Because of all the estimates that went into this calculation, the order-of-magnitude estimate $\left(10^7\,\mathrm{m}^3\right)$ is probably better to quote than the $8 \times 10^6\,\mathrm{m}^3$ figure.

†Formulas like this for volume, area, etc., are found inside the back cover of this book.

(a)



(b)

FIGURE 1–10 Example 1–6. (a) How much water is in this lake? (Photo is of one of the Rae Lakes in the Sierra Nevada of California.) (b) Model of the lake as a cylinder. [We could go one step further and estimate the mass or weight of this lake. We will see later that water has a density of $1000\,\mathrm{kg/m^3}$, so this lake has a mass of about $\left(10^3\,\mathrm{kg/m^3}\right)\left(10^7\,\mathrm{m^3}\right) \approx 10^{10}\,\mathrm{kg}$, which is about 10 billion kg or 10 million metric tons. (A metric ton is 1000 kg, about 2200 lbs, slightly larger than a British ton, 2000 lbs.)]