EXAMPLE 5–14 **Geosynchronous satellite.** A *geosynchronous* satellite is one that stays above the same point on the Earth, which is possible only if it is above a point on the equator. Such satellites are used for TV and radio transmission, for weather forecasting, and as communication relays. Determine (*a*) the height above the Earth's surface such a satellite must orbit, and (*b*) such a satellite's speed. (*c*) Compare to the speed of a satellite orbiting 200 km above Earth's surface.

PHYSICS APPLIED
Geosynchronous satellites

APPROACH To remain above the same point on Earth as the Earth rotates, the satellite must have a period of one day. We can apply Newton's second law, $F = ma$, where $a = v^2/r$ if we assume the orbit is circular.

SOLUTION (*a*) The only force on the satellite is the force of universal gravitation. So Eq. 5–4 gives us the force F, which we insert into Newton's second law:

$$F = ma$$

$$G\frac{m_{\mathrm{Sat}}\, m_{\mathrm{E}}}{r^2} = m_{\mathrm{Sat}}\frac{v^2}{r}. \qquad \text{[satellite equation]}$$

This equation has two unknowns, r and v. But the satellite revolves around the Earth with the same period that the Earth rotates on its axis, namely once in 24 hours. Thus the speed of the satellite must be

$$v = \frac{2\pi r}{T},$$

where $T = 1\text{ day} = (24\text{ h})(3600\text{ s/h}) = 86{,}400\text{ s}$. We substitute this into the "satellite equation" above and obtain (after canceling m_{Sat} on both sides)

$$G\frac{m_{\mathrm{E}}}{r^2} = \frac{(2\pi r)^2}{rT^2}.$$

After cancelling an r, we can solve for r^3:

$$r^3 = \frac{Gm_{\mathrm{E}}T^2}{4\pi^2} = \frac{(6.67 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2})(5.98 \times 10^{24}\,\mathrm{kg})(86{,}400\,\mathrm{s})^2}{4\pi^2}$$

$$= 7.54 \times 10^{22}\,\mathrm{m^3}.$$

Taking the cube root, we get $r = 4.23 \times 10^7\,\mathrm{m}$, or 42,300 km from the Earth's center. We subtract the Earth's radius of 6380 km to find that a geosynchronous satellite must orbit about 36,000 km (about $6\,r_{\mathrm{E}}$) above the Earth's surface.
(*b*) We solve for v in the satellite equation given in part (*a*):

$$v = \sqrt{\frac{Gm_{\mathrm{E}}}{r}} = \sqrt{\frac{(6.67 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2})(5.98 \times 10^{24}\,\mathrm{kg})}{(4.23 \times 10^7\,\mathrm{m})}} = 3070\,\mathrm{m/s}.$$

We get the same result if we use $v = 2\pi r/T$.
(*c*) The equation in part (*b*) for v shows $v \propto \sqrt{1/r}$. So for $r = r_{\mathrm{E}} + h = 6380\,\mathrm{km} + 200\,\mathrm{km} = 6580\,\mathrm{km}$, we get

$$v' = v\sqrt{\frac{r}{r'}} = (3070\,\mathrm{m/s})\sqrt{\frac{(42{,}300\,\mathrm{km})}{(6580\,\mathrm{km})}} = 7780\,\mathrm{m/s}.$$

NOTE The center of a satellite orbit is always at the center of the Earth; so it is not possible to have a satellite orbiting above a fixed point on the Earth at any latitude other than 0°.

EXERCISE G Two satellites orbit the Earth in circular orbits of the same radius. One satellite is twice as massive as the other. Which of the following statements is true about the speeds of these satellites? (*a*) The heavier satellite moves twice as fast as the lighter one. (*b*) The two satellites have the same speed. (*c*) The lighter satellite moves twice as fast as the heavier one. (*d*) The heavier satellite moves four times as fast as the lighter one.