5–7 Gravity Near the Earth's Surface; Geophysical Applications

When Eq. 5–4 is applied to the gravitational force between the Earth and an object at its surface, m_1 becomes the mass of the Earth m_E, m_2 becomes the mass of the object m, and r becomes the distance of the object from the Earth's center,† which is the radius of the Earth r_E. This force of gravity due to the Earth is the weight of the object, which we have been writing as mg. Thus,

$$mg = G\frac{mm_E}{r_E^2}.$$

We can solve this for g, the acceleration of gravity at the Earth's surface:

$$g = G\frac{m_E}{r_E^2}. \qquad (5\text{–}5)$$

g in terms of G

Thus, the acceleration of gravity at the surface of the Earth, g, is determined by m_E and r_E. (Don't confuse G with g; they are very different quantities, but are related by Eq. 5–5.)

CAUTION
Distinguish G from g

Until G was measured, the mass of the Earth was not known. But once G was measured, Eq. 5–5 could be used to calculate the Earth's mass, and Cavendish was the first to do so. Since $g = 9.80\,\text{m/s}^2$ and the radius of the Earth is $r_E = 6.38 \times 10^6\,\text{m}$, then, from Eq. 5–5, we obtain

$$m_E = \frac{gr_E^2}{G} = \frac{(9.80\,\text{m/s}^2)(6.38 \times 10^6\,\text{m})^2}{6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2} = 5.98 \times 10^{24}\,\text{kg}$$

Mass of the Earth

for the mass of the Earth.

Equation 5–5 can be applied to other planets, where g, m, and r would refer to that planet.

EXAMPLE 5–13 ESTIMATE Gravity on Everest. Estimate the effective value of g on the top of Mt. Everest, 8850 m (29,035 ft) above sea level. That is, what is the acceleration due to gravity of objects allowed to fall freely at this altitude?

APPROACH The force of gravity (and the acceleration due to gravity g) depends on the distance from the center of the Earth, so there will be an effective value g' on top of Mt. Everest which will be smaller than g at sea level. We assume the Earth is a uniform sphere (a reasonable "estimate").

SOLUTION We use Eq. 5–5, with r_E replaced by $r = 6380\,\text{km} + 8.9\,\text{km} = 6389\,\text{km} = 6.389 \times 10^6\,\text{m}$:

$$g = G\frac{m_E}{r^2} = \frac{(6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\,\text{kg})}{(6.389 \times 10^6\,\text{m})^2} = 9.77\,\text{m/s}^2,$$

which is a reduction of about 3 parts in a thousand (0.3%).

NOTE This is an estimate because, among other things, we ignored the mass accumulated under the mountaintop.

TABLE 5–1 Acceleration Due to Gravity at Various Locations on Earth

Location	Elevation (m)	g (m/s^2)
New York	0	9.803
San Francisco	0	9.800
Denver	1650	9.796
Pikes Peak	4300	9.789
Sydney, Australia	0	9.798
Equator	0	9.780
North Pole (calculated)	0	9.832

Note that Eq. 5–5 does not give precise values for g at different locations because the Earth is not a perfect sphere. The Earth not only has mountains and valleys, and bulges at the equator, but also its mass is not distributed precisely uniformly (see Table 5–1). The Earth's rotation also affects the value of g. However, for most practical purposes, when an object is near the Earth's surface, we will simply use $g = 9.80\,\text{m/s}^2$ and write the weight of an object as mg.

†That the distance is measured from the Earth's center does not imply that the force of gravity somehow emanates from that one point. Rather, all parts of the Earth attract gravitationally, but the net effect is a force acting toward the Earth's center.