

TABLE 1.6 Dimensions and Common Units of Area, Volume, Speed, and Acceleration

System	Area (L ²)	Volume (L ³)	Speed (L/T)	Acceleration (L/T ²)
SI	m ²	m ³	m/s	m/s ²
British engineering	ft ²	ft ³	ft/s	ft/s ²

By following these simple rules, you can use dimensional analysis to help determine whether an expression has the correct form. The relationship can be correct only if the dimensions are the same on both sides of the equation.

To illustrate this procedure, suppose you wish to derive a formula for the distance x traveled by a car in a time t if the car starts from rest and moves with constant acceleration a . In Chapter 2, we shall find that the correct expression is $x = \frac{1}{2}at^2$. Let us use dimensional analysis to check the validity of this expression. The quantity x on the left side has the dimension of length. For the equation to be dimensionally correct, the quantity on the right side must also have the dimension of length. We can perform a dimensional check by substituting the dimensions for acceleration, L/T^2 , and time, T , into the equation. That is, the dimensional form of the equation $x = \frac{1}{2}at^2$ is

$$L = \frac{L}{T^2} \cdot T^2 = L$$

The units of time squared cancel as shown, leaving the unit of length.

A more general procedure using dimensional analysis is to set up an expression of the form

$$x \propto a^n t^m$$

where n and m are exponents that must be determined and the symbol \propto indicates a proportionality. This relationship is correct only if the dimensions of both sides are the same. Because the dimension of the left side is length, the dimension of the right side must also be length. That is,

$$[a^n t^m] = L = LT^0$$

Because the dimensions of acceleration are L/T^2 and the dimension of time is T , we have

$$\left(\frac{L}{T^2}\right)^n T^m = L^1$$

$$L^n T^{m-2n} = L^1$$

Because the exponents of L and T must be the same on both sides, the dimensional equation is balanced under the conditions $m - 2n = 0$, $n = 1$, and $m = 2$. Returning to our original expression $x \propto a^n t^m$ we conclude that $x \propto at^2$. This result differs by a factor of 2 from the correct expression, which is $x = \frac{1}{2}at^2$. Because the factor $\frac{1}{2}$ is dimensionless, there is no way of determining it using dimensional analysis.