With this idea that it is the Earth's gravity that holds the Moon in its orbit, Newton developed his great theory of gravitation. But there was controversy at the time. Many thinkers had trouble accepting the idea of a force "acting at a distance." Typical forces act through contact—your hand pushes a cart and pulls a wagon, a bat hits a ball, and so on. But gravity acts without contact, said Newton: the Earth exerts a force on a falling apple and on the Moon, even though there is no contact, and the two objects may even be very far apart.

Newton set about determining the magnitude of the gravitational force that the Earth exerts on the Moon as compared to the gravitational force on objects at the Earth's surface. The centripetal acceleration of the Moon, as we calculated in Example 5–2, is $a_R = 0.00272\,\mathrm{m/s^2}$. In terms of the acceleration of gravity at the Earth's surface, $g = 9.80\,\mathrm{m/s^2}$,

The Moon's acceleration toward Earth

$$a_R = \frac{0.00272\,\mathrm{m/s^2}}{9.8\,\mathrm{m/s^2}} \approx \frac{1}{3600}\,g.$$

That is, the acceleration of the Moon toward the Earth is about $\frac{1}{3600}$ as great as the acceleration of objects at the Earth's surface. The Moon is 384,000 km from the Earth, which is about 60 times the Earth's radius of 6380 km. That is, the Moon is 60 times farther from the Earth's center than are objects at the Earth's surface. But $60 \times 60 = 60^2 = 3600$. Again that number 3600. Newton concluded that the gravitational force exerted by the Earth on any object decreases with the square of its distance r from the Earth's center:

$$\text{force of gravity} \propto \frac{1}{r^2}.$$

The Moon is 60 Earth radii away, so it feels a gravitational force only $\frac{1}{60^2} = \frac{1}{3600}$ times as strong as an equal mass would at the Earth's surface.

Newton realized that the force of gravity on an object depends not only on distance but also on the object's mass. In fact, it is directly proportional to its mass, as we have seen. According to Newton's third law, when the Earth exerts its gravitational force on any object, such as the Moon, that object exerts an equal and opposite force on the Earth (Fig. 5–19). Because of this symmetry, Newton reasoned, the magnitude of the force of gravity must be proportional to *both* the masses. Thus

$$F \propto \frac{m_E\, m_{Obj}}{r^2},$$

where m_E is the mass of the Earth, m_{Obj} the mass of the other object, and r the distance from the Earth's center to the center of the other object.

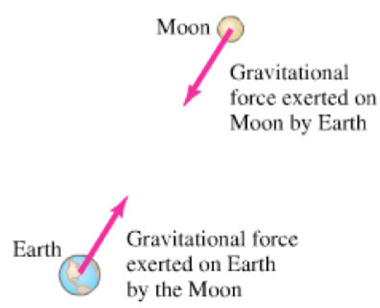

FIGURE 5–19 The gravitational force one object exerts on a second object is directed toward the first object, and (by Newton's third law) is equal and opposite to the force exerted by the second object on the first.

Newton went a step further in his analysis of gravity. In his examination of the orbits of the planets, he concluded that the force required to hold the planets in their orbits around the Sun seems to diminish as the inverse square of their distance from the Sun. This led him to believe that it is also the gravitational force that acts between the Sun and each of the planets to keep them in their orbits. And if gravity acts between these objects, why not between all objects?