PHYSICS APPLIED
Banked curves

The banking of curves can reduce the chance of skidding. The normal force exerted by a banked road, acting perpendicular to the road, will have a component toward the center of the circle (Fig. 5–14), thus reducing the reliance on friction. For a given banking angle θ, there will be one speed for which no friction at all is required. This will be the case when the horizontal component of the normal force toward the center of the curve, $F_N \sin\theta$ (see Fig. 5–14), is just equal to the force required to give a vehicle its centripetal acceleration—that is, when

$$F_N \sin\theta = m\frac{v^2}{r}. \qquad \text{[no friction required]}$$

The banking angle of a road, θ, is chosen so that this condition holds for a particular speed, called the "design speed."

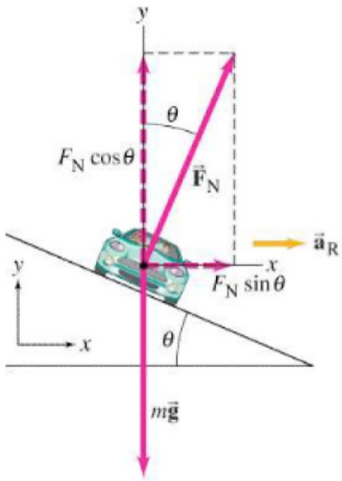

FIGURE 5–14 Normal force on a car rounding a banked curve, resolved into its horizontal and vertical components. The centripetal acceleration is horizontal (*not* parallel to the sloping road). The friction force on the tires, not shown, could point up or down along the slope, depending on the car's speed. The friction force will be zero for one particular speed.

EXAMPLE 5–7 **Banking angle.** (*a*) For a car traveling with speed v around a curve of radius r, determine a formula for the angle at which a road should be banked so that no friction is required. (*b*) What is this angle for an expressway off-ramp curve of radius 50 m at a design speed of 50 km/h?

APPROACH Even though the road is banked, the car is still moving along a horizontal circle, so the centripetal acceleration needs to be horizontal. We choose our x and y axes as horizontal and vertical so that a_R, which is horizontal, is along the x axis. The forces on the car are the Earth's gravity mg downward, and the normal force F_N exerted by the road perpendicular to its surface. See Fig. 5–14, where the components of F_N are also shown. We don't need to consider the friction of the road because we are designing a road to be banked so as to eliminate dependence on friction.

CAUTION
F_N is not always equal to mg

SOLUTION (*a*) For the horizontal direction, $\Sigma F_R = ma_R$ gives

$$F_N \sin\theta = \frac{mv^2}{r}.$$

Since there is no vertical motion, the y component of the acceleration is zero, so $\Sigma F_y = ma_y$ gives us

$$F_N \cos\theta - mg = 0.$$

Thus,

$$F_N = \frac{mg}{\cos\theta}.$$

[Note in this case that $F_N \geq mg$ since $\cos\theta \leq 1$.]
We substitute this relation for F_N into the equation for the horizontal motion,

Horizontal component of normal force acts to provide centripetal acceleration (friction is desired to be zero—otherwise it too would contribute)

$$F_N \sin\theta = m\frac{v^2}{r},$$

and obtain

$$\frac{mg}{\cos\theta}\sin\theta = m\frac{v^2}{r}$$

or

$$mg\tan\theta = m\frac{v^2}{r},$$

so

Banking angle (friction not needed)

$$\tan\theta = \frac{v^2}{rg}.$$

This is the formula for the banking angle θ: no friction needed at speed v.
(*b*) For $r = 50$ m and $v = 50$ km/h (or 14 m/s),

$$\tan\theta = \frac{(14\text{ m/s})^2}{(50\text{ m})(9.8\text{ m/s}^2)} = 0.40,$$

so $\theta = 22°$.