EXAMPLE 5–4 **Revolving ball (vertical circle).** A 0.150-kg ball on the end of a 1.10-m-long cord (negligible mass) is swung in a *vertical* circle. (*a*) Determine the minimum speed the ball must have at the top of its arc so that the ball continues moving in a circle. (*b*) Calculate the tension in the cord at the bottom of the arc, assuming the ball is moving at twice the speed of part (*a*).

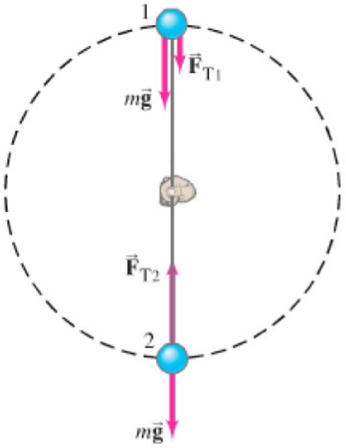


FIGURE 5–8 Example 5–4. Free-body diagrams for positions 1 and 2.

APPROACH The ball moves in a vertical circle and is *not* undergoing uniform circular motion. The radius is assumed constant, but the speed v changes because of gravity. Nonetheless, Eq. 5–1 is valid at each point along the circle, and we use it at points 1 and 2. The free-body diagram is shown in Fig. 5–8 for both positions 1 and 2.

SOLUTION (*a*) At the top (point 1), two forces act on the ball: $m\vec{\mathbf{g}}$, the force of gravity, and $\vec{\mathbf{F}}_{T1}$, the tension force the cord exerts at point 1. Both act downward, and their vector sum acts to give the ball its centripetal acceleration a_R. We apply Newton's second law, for the vertical direction, choosing downward as positive since the acceleration is downward (toward the center):

$$(\Sigma F)_R = ma_R$$

$$F_{T1} + mg = m\frac{v_1^2}{r}. \qquad \text{[at top]}$$

Cord tension and gravity together provide centripetal acceleration

From this equation we can see that the tension force F_{T1} at point 1 will get larger if v_1 (ball's speed at top of circle) is made larger, as expected. But we are asked for the *minimum* speed to keep the ball moving in a circle. The cord will remain taut as long as there is tension in it. But if the tension disappears (because v_1 is too small) the cord can go limp, and the ball will fall out of its circular path. Thus, the minimum speed will occur if $F_{T1} = 0$, for which we have

$$mg = m\frac{v_1^2}{r}. \qquad \text{[minimum speed at top]}$$

Gravity provides centripetal acceleration

We solve for v_1:

$$v_1 = \sqrt{gr} = \sqrt{(9.80\,\text{m/s}^2)(1.10\,\text{m})} = 3.28\,\text{m/s}.$$

This is the minimum speed at the top of the circle if the ball is to continue moving in a circular path.

(*b*) When the ball is at the bottom of the circle (point 2 in Fig. 5–8), the cord exerts its tension force F_{T2} upward, whereas the force of gravity, $m\vec{\mathbf{g}}$, still acts downward. So we apply Newton's second law, this time choosing *upward* as positive since the acceleration is upward (toward the center):

$$(\Sigma F)_R = ma_R$$

$$F_{T2} - mg = m\frac{v_2^2}{r}. \qquad \text{[at bottom]}$$

String tension and gravity acting in opposite directions provide centripetal acceleration

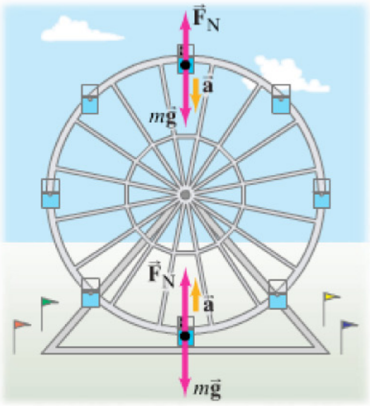
The speed v_2 is given as twice that in (*a*), namely 6.56 m/s. We solve for F_{T2}:

$$F_{T2} = m\frac{v_2^2}{r} + mg$$

$$= (0.150\,\text{kg})\frac{(6.56\,\text{m/s})^2}{(1.10\,\text{m})} + (0.150\,\text{kg})(9.80\,\text{m/s}^2) = 7.34\,\text{N}.$$

EXERCISE B In a tumble dryer, the speed of the drum should be just large enough so that the clothes are carried nearly to the top of the drum and then fall away, rather than being pressed against the drum for the whole revolution. Determine whether this speed will be different for heavier wet clothes than for lighter dry clothes.

FIGURE 5–9 Exercise C.



EXERCISE C A rider on a Ferris wheel moves in a vertical circle of radius r at constant speed v (Fig. 5–9). Is the normal force that the seat exerts on the rider at the top of the wheel (*a*) less than, (*b*) more than, or (*c*) the same as, the force the seat exerts at the bottom of the wheel?