

You must be able to easily alternate between the linear and rotational variables that describe a given situation.

The **moment of inertia of a system of particles** is

$$I \equiv \sum_i m_i r_i^2 \quad (10.15)$$

If a rigid object rotates about a fixed axis with angular speed ω , its **rotational energy** can be written

$$K_R = \frac{1}{2} I \omega^2 \quad (10.16)$$

where I is the moment of inertia about the axis of rotation.

The **moment of inertia of a rigid object** is

$$I = \int r^2 dm \quad (10.17)$$

where r is the distance from the mass element dm to the axis of rotation.

The magnitude of the **torque** associated with a force \mathbf{F} acting on an object is

$$\tau = Fd \quad (10.19)$$

where d is the moment arm of the force, which is the perpendicular distance from some origin to the line of action of the force. Torque is a measure of the tendency of the force to change the rotation of the object about some axis.

If a rigid object free to rotate about a fixed axis has a **net external torque** acting on it, the object undergoes an angular acceleration α , where

$$\sum \tau = I\alpha \quad (10.21)$$

The rate at which work is done by an external force in rotating a rigid object about a fixed axis, or the **power** delivered, is

$$\mathcal{P} = \tau\omega \quad (10.23)$$

The net work done by external forces in rotating a rigid object about a fixed axis equals the change in the rotational kinetic energy of the object:

$$\sum W = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2 \quad (10.24)$$

QUESTIONS

- What is the angular speed of the second hand of a clock? What is the direction of $\boldsymbol{\omega}$ as you view a clock hanging vertically? What is the magnitude of the angular acceleration vector $\boldsymbol{\alpha}$ of the second hand?
- A wheel rotates counterclockwise in the xy plane. What is the direction of $\boldsymbol{\omega}$? What is the direction of $\boldsymbol{\alpha}$ if the angular velocity is decreasing in time?
- Are the kinematic expressions for θ , ω , and α valid when the angular displacement is measured in degrees instead of in radians?
- A turntable rotates at a constant rate of 45 rev/min. What is its angular speed in radians per second? What is the magnitude of its angular acceleration?
- Suppose $a = b$ and $M > m$ for the system of particles described in Figure 10.8. About what axis (x , y , or z) does the moment of inertia have the smallest value? the largest value?
- Suppose the rod in Figure 10.10 has a nonuniform mass distribution. In general, would the moment of inertia about the y axis still equal $ML^2/12$? If not, could the moment of inertia be calculated without knowledge of the manner in which the mass is distributed?
- Suppose that only two external forces act on a rigid body, and the two forces are equal in magnitude but opposite in direction. Under what condition does the body rotate?
- Explain how you might use the apparatus described in Example 10.12 to determine the moment of inertia of the wheel. (If the wheel does not have a uniform mass density, the moment of inertia is not necessarily equal to $\frac{1}{2}MR^2$.)