**List of equations in classical mechanics**

[Classical mechanics](http://en.wikipedia.org/wiki/Classical_mechanics) is the branch of [physics](http://en.wikipedia.org/wiki/Physics) used to describe the motion of [macroscopic](http://en.wikipedia.org/wiki/Macroscopic) objects.[[1]](http://en.wikipedia.org/wiki/List_of_equations_in_classical_mechanics#cite_note-1) It is the most familiar of the theories of physics. The concepts it covers, such as [mass](http://en.wikipedia.org/wiki/Mass), [acceleration](http://en.wikipedia.org/wiki/Acceleration), and [force](http://en.wikipedia.org/wiki/Force), are commonly used and known.[[2]](http://en.wikipedia.org/wiki/List_of_equations_in_classical_mechanics#cite_note-2) The subject is based upon a [three-dimensional](http://en.wikipedia.org/wiki/Three-dimensional_space) [Euclidean space](http://en.wikipedia.org/wiki/Euclidean_space) with fixed axes, called a frame of reference. The point of [concurrency](http://en.wikipedia.org/wiki/Concurrent_lines) of the three axes is known as the origin of the particular space.[[3]](http://en.wikipedia.org/wiki/List_of_equations_in_classical_mechanics#cite_note-3)

Classical mechanics utilises many [equations](http://en.wikipedia.org/wiki/Equation)—as well as other [mathematical](http://en.wikipedia.org/wiki/Mathematics) concepts—which relate various physical quantities to one another. These include [differential equations](http://en.wikipedia.org/wiki/Differential_equations), [manifolds](http://en.wikipedia.org/wiki/Manifold), [Lie groups](http://en.wikipedia.org/wiki/Lie_group), and [ergodic theory](http://en.wikipedia.org/wiki/Ergodic_theory).[[4]](http://en.wikipedia.org/wiki/List_of_equations_in_classical_mechanics#cite_note-4) This page gives a summary of the most important of these.

This article lists equations from [Newtonian mechanics](http://en.wikipedia.org/wiki/Newtonian_mechanics), see [analytical mechanics](http://en.wikipedia.org/wiki/Analytical_mechanics) for the more general formulation of classical mechanics (which includes [Lagrangian](http://en.wikipedia.org/wiki/Lagrangian_mechanics) and [Hamiltonian mechanics](http://en.wikipedia.org/wiki/Hamiltonian_mechanics)).

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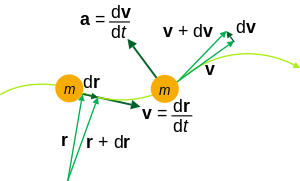
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**Classical mechanics**

**Mass and inertia**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Quantity (common name/s)** | **(Common) symbol/s** | **Defining equation** | **SI units** | **Dimension** |
| Linear, surface, volumetric mass density | *λ* or *μ* (especially in [acoustics](http://en.wikipedia.org/wiki/Acoustics), see below) for Linear, *σ* for surface, *ρ* for volume. | m = \int \lambda \mathrm{d} \ell  m = \iint \sigma \mathrm{d} S  m = \iiint \rho \mathrm{d} V \,\! | kg m−*n*, *n* = 1, 2, 3 | [M][L]−*n* |
| Moment of mass[[5]](http://en.wikipedia.org/wiki/List_of_equations_in_classical_mechanics#cite_note-5) | **m** (No common symbol) | Point mass:  \mathbf{m} = \mathbf{r}m \,\!  Discrete masses about an axis  x_i \,\!:  \mathbf{m} = \sum_{i=1}^N \mathbf{r}_\mathrm{i} m_i \,\!  Continuum of mass about an axis  x_i \,\!:  \mathbf{m} = \int \rho \left ( \mathbf{r} \right ) x_i \mathrm{d} \mathbf{r} \,\! | kg m | [M][L] |
| [Centre of mass](http://en.wikipedia.org/wiki/Centre_of_mass) | **r**com  (Symbols vary) | *i*th moment of mass  \mathbf{m}_\mathrm{i} = \mathbf{r}_\mathrm{i} m_i \,\!  Discrete masses:  \mathbf{r}_\mathrm{com} = \frac{1}{M}\sum_i \mathbf{r}_\mathrm{i} m_i = \frac{1}{M}\sum_i \mathbf{m}_\mathrm{i} \,\!  Mass continuum:  \mathbf{r}_\mathrm{com} = \frac{1}{M}\int \mathrm{d}\mathbf{m} = \frac{1}{M}\int \mathbf{r} \mathrm{d}m = \frac{1}{M}\int \mathbf{r} \rho \mathrm{d}V \,\! | m | [L] |
| 2-Body reduced mass | *m*12, *μ* Pair of masses = *m*1 and *m*2 | \mu = \left (m_1m_2 \right )/\left ( m_1 + m_2 \right) \,\! | kg | [M] |
| Moment of inertia (MOI) | *I* | Discrete Masses:  I = \sum_i \mathbf{m}_\mathrm{i} \cdot \mathbf{r}_\mathrm{i} = \sum_i \left | \mathbf{r}_\mathrm{i} \right | ^2 m \,\!  Mass continuum:  I = \int \left | \mathbf{r} \right | ^2 \mathrm{d} m = \int \mathbf{r} \cdot \mathrm{d} \mathbf{m}  = \int \left | \mathbf{r} \right | ^2 \rho \mathrm{d}V \,\! | kg m2 | [M][L]2 |

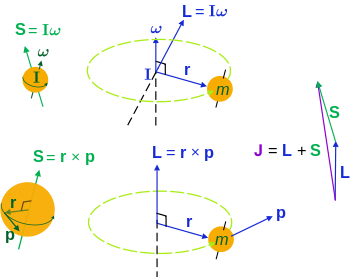
**Derived kinematic quantities**

[](http://en.wikipedia.org/wiki/File:Kinematics.svg)

Kinematic quantities of a classical particle: mass *m*, position **r**, velocity **v**, acceleration **a**.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Quantity (common name/s)** | **(Common) symbol/s** | **Defining equation** | **SI units** | **Dimension** |
| [Velocity](http://en.wikipedia.org/wiki/Velocity) | **v** | \mathbf{v} = \mathrm{d} \mathbf{r}/\mathrm{d} t \,\! | m s−1 | [L][T]−1 |
| [Acceleration](http://en.wikipedia.org/wiki/Acceleration) | **a** | \mathbf{a} = \mathrm{d} \mathbf{v}/\mathrm{d} t = \mathrm{d}^2 \mathbf{r}/\mathrm{d} t^2  \,\! | m s−2 | [L][T]−2 |
| [Jerk](http://en.wikipedia.org/wiki/Jerk_%28physics%29) | **j** | \mathbf{j} = \mathrm{d} \mathbf{a}/\mathrm{d} t = \mathrm{d}^3 \mathbf{r}/\mathrm{d} t^3 \,\! | m s−3 | [L][T]−3 |
| [Angular velocity](http://en.wikipedia.org/wiki/Angular_velocity) | **ω** | \boldsymbol{\omega} = \mathbf{\hat{n}} \left ( \mathrm{d} \theta /\mathrm{d} t \right ) \,\! | rad s−1 | [T]−1 |
| [Angular Acceleration](http://en.wikipedia.org/wiki/Angular_acceleration) | **α** | \boldsymbol{\alpha} = \mathrm{d} \boldsymbol{\omega}/\mathrm{d} t = \mathbf{\hat{n}} \left ( \mathrm{d}^2 \theta / \mathrm{d} t^2 \right ) \,\! | rad s−2 | [T]−2 |

**Derived dynamic quantities**

[](http://en.wikipedia.org/wiki/File:Classical_angular_momentum.svg)

Angular momenta of a classical object.  
  
**Left:** intrinsic "spin" angular momentum **S** is really orbital angular momentum of the object at every point,  
  
**right:** extrinsic orbital angular momentum **L** about an axis,  
  
**top:** the [moment of inertia tensor](http://en.wikipedia.org/wiki/Moment_of_inertia_tensor) **I** and angular velocity **ω** (**L** is not always parallel to **ω**)[[6]](http://en.wikipedia.org/wiki/List_of_equations_in_classical_mechanics#cite_note-6)  
  
**bottom:** momentum **p** and it's radial position **r** from the axis.  
  
The total angular momentum (spin + orbital) is **J**.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Quantity (common name/s)** | **(Common) symbol/s** | **Defining equation** | **SI units** | **Dimension** |
| [Momentum](http://en.wikipedia.org/wiki/Momentum) | **p** | \mathbf{p}=m\mathbf{v} \,\! | kg m s−1 | [M][L][T]−1 |
| [Force](http://en.wikipedia.org/wiki/Force) | **F** | \mathbf{F} = \mathrm{d} \mathbf{p}/\mathrm{d} t \,\! | N = kg m s−2 | [M][L][T]−2 |
| [Impulse](http://en.wikipedia.org/wiki/Impulse_%28physics%29) | Δ**p**, **I** | \mathbf{I} = \Delta \mathbf{p} = \int_{t_1}^{t_2} \mathbf{F}\mathrm{d} t \,\! | kg m s−1 | [M][L][T]−1 |
| [Angular momentum](http://en.wikipedia.org/wiki/Angular_momentum) about a position point **r**0, | **L**, **J**, **S** | \mathbf{L} = \left ( \mathbf{r} - \mathbf{r}_0 \right ) \times \mathbf{p} \,\!  Most of the time we can set **r**0 = **0** if particles are orbiting about axes intersecting at a common point. | kg m2 s−1 | [M][L]2[T]−1 |
| Moment of a force about a position point **r**0,  [Torque](http://en.wikipedia.org/wiki/Torque) | **τ**, **M** | \boldsymbol{\tau} = \left ( \mathbf{r} - \mathbf{r}_0 \right ) \times \mathbf{F} = \mathrm{d} \mathbf{L}/\mathrm{d} t \,\! | N m = kg m2 s−2 | [M][L]2[T]−2 |
| Angular impulse | Δ**L** (no common symbol) | \Delta \mathbf{L} = \int_{t_1}^{t_2} \boldsymbol{\tau}\mathrm{d} t \,\! | kg m2 s−1 | [M][L]2[T]−1 |

**General energy definitions**

Main article: [Mechanical energy](http://en.wikipedia.org/wiki/Mechanical_energy)

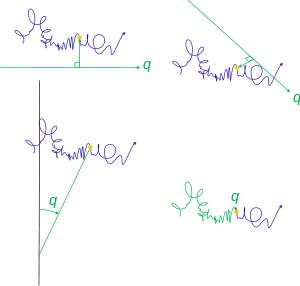
|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Quantity (common name/s)** | **(Common) symbol/s** | **Defining equation** | **SI units** | **Dimension** |
| [Mechanical work](http://en.wikipedia.org/wiki/Work_%28physics%29) due  to a Resultant Force | *W* | W = \int_C \mathbf{F} \cdot \mathrm{d} \mathbf{r} \,\! | J = N m = kg m2 s−2 | [M][L]2[T]−2 |
| Work done ON mechanical  system, Work done BY | *W*ON, *W*BY | \Delta W_\mathrm{ON} = - \Delta W_\mathrm{BY} \,\! | J = N m = kg m2 s−2 | [M][L]2[T]−2 |
| [Potential energy](http://en.wikipedia.org/wiki/Potential_energy) | *φ, Φ, U, V, Ep* | \Delta W = - \Delta V \,\! | J = N m = kg m2 s−2 | [M][L]2[T]−2 |
| Mechanical [power](http://en.wikipedia.org/wiki/Power_%28physics%29) | *P* | P = \mathrm{d}E/\mathrm{d}t \,\! | W = J s−1 | [M][L]2[T]−3 |

Every [conservative force](http://en.wikipedia.org/wiki/Conservative_force) has a [potential energy](http://en.wikipedia.org/wiki/Potential_energy). By following two principles one can consistently assign a non-relative value to *U*:

* Wherever the force is zero, its potential energy is defined to be zero as well.
* Whenever the force does work, potential energy is lost.

**Generalized mechanics**

Main articles: [Analytical mechanics](http://en.wikipedia.org/wiki/Analytical_mechanics), [Lagrangian mechanics](http://en.wikipedia.org/wiki/Lagrangian_mechanics) and [Hamiltonian mechanics](http://en.wikipedia.org/wiki/Hamiltonian_mechanics)

[](http://en.wikipedia.org/wiki/File:Generalized_coordinates_1df.svg)

[Generalized coordinates](http://en.wikipedia.org/wiki/Generalized_coordinates) for one degree of freedom (of a particle moving in a complicated path). Instead of using all three [Cartesian coordinates](http://en.wikipedia.org/wiki/Cartesian_coordinates) *x, y, z* (or other standard [coordinate systems](http://en.wikipedia.org/wiki/Coordinate_systems)), only one is needed and is completely arbitrary to define the position. Four possibilities are shown.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Quantity (common name/s)** | **(Common) symbol/s** | **Defining equation** | **SI units** | **Dimension** |
| [Generalized coordinates](http://en.wikipedia.org/wiki/Generalized_coordinates) | *q, Q* |  | varies with choice | varies with choice |
| [Generalized velocities](http://en.wikipedia.org/wiki/Generalized_velocities) | \dot{q},\dot{Q} \,\! | \dot{q}\equiv \mathrm{d}q/\mathrm{d}t \,\! | varies with choice | varies with choice |
| [Generalized momenta](http://en.wikipedia.org/wiki/Canonical_coordinates) | *p, P* | p = \partial L /\partial \dot{q} \,\! | varies with choice | varies with choice |
| [Lagrangian](http://en.wikipedia.org/wiki/Lagrangian) | *L* | L(\mathbf{q},\mathbf{\dot{q}},t) = T(\mathbf{\dot{q}})-V(\mathbf{q},\mathbf{\dot{q}},t) \,\!  where  \mathbf{q}=\mathbf{q}(t) \,\!and **p** = **p**(*t*) are vectors of the generalized coords and momenta, as functions of time | J | [M][L]2[T]−2 |
| [Hamiltonian](http://en.wikipedia.org/wiki/Hamiltonian_mechanics) | *H* | H(\mathbf{p},\mathbf{q},t) = \mathbf{p}\cdot\mathbf{\dot{q}} - L(\mathbf{q},\mathbf{\dot{q}},t) \,\! | J | [M][L]2[T]−2 |
| [Action](http://en.wikipedia.org/wiki/Action_%28physics%29), Hamilton's principle function | *S*,  \scriptstyle{\mathcal{S}} \,\! | \mathcal{S} = \int_{t_1}^{t_2} L(\mathbf{q},\mathbf{\dot{q}},t) \mathrm{d}t \,\! | J s | [M][L]2[T]−1 |

**Kinematics**

In the following rotational definitions, the angle can be any angle about the specified axis of rotation. It is customary to use *θ*, but this does not have to be the polar angle used in polar coordinate systems. The unit axial vector

\bold{\hat{n}} = \bold{\hat{e}}_r\times\bold{\hat{e}}_\theta \,\!

defines the axis of rotation,  \scriptstyle \bold{\hat{e}}_r \,\!= unit vector in direction of **r**,  \scriptstyle \bold{\hat{e}}_\theta \,\!= unit vector tangential to the angle.

|  |  |  |
| --- | --- | --- |
|  | **Translation** | **Rotation** |
| [**Velocity**](http://en.wikipedia.org/wiki/Velocity) | Average:  \mathbf{v}_{\mathrm{average}} = {\Delta \mathbf{r} \over \Delta t}  Instantaneous:  \mathbf{v} = {d\mathbf{r} \over dt} | [Angular velocity](http://en.wikipedia.org/wiki/Angular_velocity)  \boldsymbol{\omega} = \bold{\hat{n}}\frac{{\rm d} \theta}{{\rm d} t}\,\!  Rotating [rigid body](http://en.wikipedia.org/wiki/Rigid_body):  \mathbf{v} = \boldsymbol{\omega} \times \mathbf{r} \,\! |
| [**Acceleration**](http://en.wikipedia.org/wiki/Acceleration) | Average:  \mathbf{a}_{\mathrm{average}} = \frac{\Delta\mathbf{v}}{\Delta t}  Instantaneous:  \mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2} | [Angular acceleration](http://en.wikipedia.org/wiki/Angular_acceleration)  \boldsymbol{\alpha} = \frac{{\rm d} \boldsymbol{\omega}}{{\rm d} t} = \bold{\hat{n}}\frac{{\rm d}^2 \theta}{{\rm d} t^2} \,\!  Rotating rigid body:  \mathbf{a} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times \mathbf{v} \,\! |
| [**Jerk**](http://en.wikipedia.org/wiki/Jerk_%28physics%29) | Average:  \mathbf{j}_{\mathrm{average}} = \frac{\Delta\mathbf{a}}{\Delta t}  Instantaneous:  \mathbf{j} = \frac{d\mathbf{a}}{dt} = \frac{d^2\mathbf{v}}{dt^2} = \frac{d^3\mathbf{r}}{dt^3} | [Angular jerk](http://en.wikipedia.org/w/index.php?title=Angular_jerk&action=edit&redlink=1)  \boldsymbol{\zeta} = \frac{{\rm d} \boldsymbol{\alpha}}{{\rm d} t} = \bold{\hat{n}}\frac{{\rm d}^2 \omega}{{\rm d} t^2} = \bold{\hat{n}}\frac{{\rm d}^3 \theta}{{\rm d} t^3} \,\!  Rotating rigid body:  \mathbf{j} = \boldsymbol{\zeta} \times \mathbf{r} + \boldsymbol{\alpha} \times \mathbf{a} \,\! |

**Dynamics**

|  |  |  |
| --- | --- | --- |
|  | **Translation** | **Rotation** |
| [**Momentum**](http://en.wikipedia.org/wiki/Momentum) | Momentum is the "amount of translation"  \mathbf{p} = m\mathbf{v}  For a rotating rigid body:  \mathbf{p} = \boldsymbol{\omega} \times \mathbf{m} \,\! | [Angular momentum](http://en.wikipedia.org/wiki/Angular_momentum)  Angular momentum is the "amount of rotation":  \mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{I} \cdot \boldsymbol{\omega}  and the cross-product is a [pseudovector](http://en.wikipedia.org/wiki/Pseudovector) i.e. if **r** and **p** are reversed in direction (negative), **L** is not.  In general **I** is an order-2 [tensor](http://en.wikipedia.org/wiki/Tensor), see above for its components. The dot **·** indicates [tensor contraction](http://en.wikipedia.org/wiki/Tensor_contraction). |
| [**Force**](http://en.wikipedia.org/wiki/Force) **and** [**Newton's 2nd law**](http://en.wikipedia.org/wiki/Newton%27s_2nd_law) | Resultant force acts on a system at the center of mass, equal to the rate of change of momentum:  \begin{align} \mathbf{F} & = \frac{d\mathbf{p}}{dt} = \frac{d(m\mathbf{v})}{dt} \\ & = m\mathbf{a} + \mathbf{v}\frac{{\rm d}m}{{\rm d}t} \\ \end{align} \,\!  For a number of particles, the equation of motion for one particle *i* is:[[7]](http://en.wikipedia.org/wiki/List_of_equations_in_classical_mechanics#cite_note-7)  \frac{\mathrm{d}\mathbf{p}_i}{\mathrm{d}t} = \mathbf{F}_{E} + \sum_{i \neq j} \mathbf{F}_{ij} \,\!  where **p***i* = momentum of particle *i*, **F***ij* = force ***on*** particle *i* ***by*** particle *j*, and **F***E* = resultant external force (due to any agent not part of system). Particle *i* does not exert a force on itself. | [Torque](http://en.wikipedia.org/wiki/Torque)  Torque **τ** is also called moment of a force, because it is the rotational analogue to force:[[8]](http://en.wikipedia.org/wiki/List_of_equations_in_classical_mechanics#cite_note-8)  \boldsymbol{\tau} = \frac{{\rm d}\mathbf{L}}{{\rm d}t} = \mathbf{r}\times\mathbf{F} = \frac{{\rm d}(\mathbf{I} \cdot \boldsymbol{\omega})}{{\rm d}t} \,\!  For rigid bodies, Newton's 2nd law for rotation takes the same form as for translation:  \begin{align}  \boldsymbol{\tau} & = \frac{{\rm d}\bold{L}}{{\rm d}t} = \frac{{\rm d}(\bold{I}\cdot\boldsymbol{\omega})}{{\rm d}t} \\ & = \frac{{\rm d}\bold{I}}{{\rm d}t}\cdot\boldsymbol{\omega} + \bold{I}\cdot\boldsymbol{\alpha} \\ \end{align} \,\!  Likewise, for a number of particles, the equation of motion for one particle *i* is:[[9]](http://en.wikipedia.org/wiki/List_of_equations_in_classical_mechanics#cite_note-9)  \frac{\mathrm{d}\mathbf{L}_i}{\mathrm{d}t} = \boldsymbol{\tau}_E + \sum_{i \neq j} \boldsymbol{\tau}_{ij} \,\! |
| [**Yank**](http://en.wikipedia.org/w/index.php?title=Yank_%28physics%29&action=edit&redlink=1) | Yank is rate of change of force:  \begin{align} \mathbf{Y} & = \frac{d\mathbf{F}}{dt}  = \frac{d^2\mathbf{p}}{dt^2} = \frac{d^2(m\mathbf{v})}{dt^2} \\ & = m\mathbf{j} + \mathbf{2a}\frac{{\rm d}m}{{\rm d}t} + \mathbf{v}\frac{{\rm d^2}m}{{\rm d}t^2} \\ \end{align} \,\!  For constant mass, it becomes;  \mathbf{Y} = m\mathbf{j} | [Rotatum](http://en.wikipedia.org/wiki/Rotatum)  Rotatum **Ρ** is also called moment of a Yank, becuause it is the rotational analogue to yank:  \boldsymbol{\Rho} = \frac{{\rm d}\mathbf{\tau}}{{\rm d}t} = \mathbf{r}\times\mathbf{Y} = \frac{{\rm d}(\mathbf{I} \cdot \boldsymbol{\alpha})}{{\rm d}t} \,\! |
| [**Impulse**](http://en.wikipedia.org/wiki/Impulse_%28physics%29) | Impulse is the change in momentum:  \Delta \mathbf{p} = \int \mathbf{F} dt  For constant force **F**:  \Delta \mathbf{p} = \mathbf{F} \Delta t | Angular impulse is the change in angular momentum:  \Delta \mathbf{L} = \int \boldsymbol{\tau} dt  For constant torque **τ**:  \Delta \mathbf{L} = \boldsymbol{\tau} \Delta t |

**Precession**

The precession angular speed of a [spinning top](http://en.wikipedia.org/wiki/Spinning_top) is given by:

 \boldsymbol{\Omega} = \frac{wr}{I\boldsymbol{\omega}} 

where *w* is the weight of the spinning flywheel.

**Energy**

The mechanical work done by an external agent on a system is equal to the change in kinetic energy of the system:

General [work-energy theorem](http://en.wikipedia.org/wiki/Work-energy_theorem) (translation and rotation)

The work done *W* by an external agent which exerts a force **F** (at **r**) and torque **τ** on an object along a curved path *C* is:

  W = \Delta T = \int_C \left ( \mathbf{F} \cdot \mathrm{d} \mathbf{r} + \boldsymbol{\tau} \cdot \mathbf{n} {\mathrm{d} \theta} \right ) \,\!

where θ is the angle of rotation about an axis defined by a [unit vector](http://en.wikipedia.org/wiki/Unit_vector) **n**.

[Kinetic energy](http://en.wikipedia.org/wiki/Kinetic_energy)

 \Delta E_k = W = \frac{1}{2} m(v^2 - {v_0}^2) 

[Elastic potential energy](http://en.wikipedia.org/wiki/Elastic_potential_energy)

For a stretched spring fixed at one end obeying [Hooke's law](http://en.wikipedia.org/wiki/Hooke%27s_law):

 \Delta E_p =  \frac{1}{2} k(r_2-r_1)^2 \,\!

where *r*2 and *r*1 are collinear coordinates of the free end of the spring, in the direction of the extension/compression, and k is the spring constant.

**Euler's equations for rigid body dynamics**

Main article: [Euler's equations (rigid body dynamics)](http://en.wikipedia.org/wiki/Euler%27s_equations_%28rigid_body_dynamics%29)

[Euler](http://en.wikipedia.org/wiki/Euler) also worked out analogous laws of motion to those of Newton, see [Euler's laws of motion](http://en.wikipedia.org/wiki/Euler%27s_laws_of_motion). These extend the scope of Newton's laws to rigid bodies, but are essentially the same as above. A new equation Euler formulated is:[[10]](http://en.wikipedia.org/wiki/List_of_equations_in_classical_mechanics" \l "cite_note-10)

 \mathbf{I} \cdot \boldsymbol{\alpha} + \boldsymbol{\omega} \times \left ( \mathbf{I} \cdot \boldsymbol{\omega} \right ) = \boldsymbol{\tau} \,\!

where **I** is the [moment of inertia](http://en.wikipedia.org/wiki/Moment_of_inertia) [tensor](http://en.wikipedia.org/wiki/Tensor).

**General planar motion**

See also: [Polar coordinate system (section: vector calculus)](http://en.wikipedia.org/wiki/Polar_coordinate_system#Vector_calculus)

The previous equations for planar motion can be used here: corollaries of momentum, angular momentum etc. can immediately follow by applying the above definitions. For any object moving in any path in a plane,

 \mathbf{r}= \bold{r}(t) = r\bold{\hat{e}}_r  \,\!

the following general results apply to the particle.

|  |  |
| --- | --- |
| **Kinematics** | **Dynamics** |
| Position  \mathbf{r} =\bold{r}\left ( r,\theta, t \right ) = r \bold{\hat{e}}_r |  |
| Velocity  \mathbf{v} = \bold{\hat{e}}_r \frac{\mathrm{d} r}{\mathrm{d}t} + r \omega \bold{\hat{e}}_\theta | Momentum  \mathbf{p} = m \left(\bold{\hat{e}}_r \frac{\mathrm{d}^2 r}{\mathrm{d}t^2} + r \omega \bold{\hat{e}}_\theta \right)  Angular momenta \mathbf{L} = m \bold{r}\times \left(\bold{\hat{e}}_r \frac{\mathrm{d}^2 r}{\mathrm{d}t^2} + r \omega \bold{\hat{e}}_\theta \right) |
| Acceleration  \mathbf{a} =\left ( \frac{\mathrm{d}^2 r}{\mathrm{d}t^2} - r\omega^2\right )\bold{\hat{e}}_r + \left ( r \alpha + 2 \omega \frac{\mathrm{d}r}{{\rm d}t} \right )\bold{\hat{e}}_\theta | The [centripetal force](http://en.wikipedia.org/wiki/Centripetal_force) is  \mathbf{F}_\bot = - m \omega^2 R \bold{\hat{e}}_r= - \omega^2 \mathbf{m} \,\!  where again **m** is the mass moment, and the [coriolis force](http://en.wikipedia.org/wiki/Coriolis_force) is  \mathbf{F}_c = 2\omega \frac{{\rm d}r}{{\rm d}t} \bold{\hat{e}}_\theta = 2\omega v \bold{\hat{e}}_\theta \,\!  The [Coriolis acceleration and force](http://en.wikipedia.org/wiki/Coriolis_effect) can also be written:  \mathbf{F}_c = m\mathbf{a}_c = -2 m \boldsymbol{ \omega \times v} |

**Central force motion**

For a massive body moving in a [central potential](http://en.wikipedia.org/wiki/Central_potential) due to another object, which depends only on the radial separation between the centres of masses of the two objects, the equation of motion is:

\frac{d^2}{d\theta^2}\left(\frac{1}{\mathbf{r}}\right) + \frac{1}{\mathbf{r}} = -\frac{\mu\mathbf{r}^2}{\mathbf{l}^2}\mathbf{F}(\mathbf{r})

**Equations of motion (constant acceleration)**

These equations can be used only when acceleration is constant. If acceleration is not constant then the general [calculus](http://en.wikipedia.org/wiki/Calculus) equations above must be used, found by integrating the definitions of position, velocity and acceleration (see above).

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| **Linear motion** | **Angular motion** |
| v = v_0+at \, | \omega _1 = \omega _0 + \alpha t \, |
| s = \frac {1} {2}(v_0+v) t | \theta = \frac{1}{2}(\omega _0 + \omega _1)t |
| s = v_0 t + \frac {1} {2} a t^2 | \theta = \omega _0 t + \frac{1}{2} \alpha t^2 |
| v^2 = v_0^2 + 2 a s \, | \omega _1^2 = \omega _0^2 + 2\alpha\theta |
| s = v t - \frac{1}{2} a t^2 | \theta = \omega _1 t - \frac{1}{2} \alpha t^2 |

See also: [Linear motion § Analogy between linear and rotational motion](http://en.wikipedia.org/wiki/Linear_motion#Analogy_between_linear_and_rotational_motion)

**Galilean frame transforms**

For classical (Galileo-Newtonian) mechanics, the transformation law from one inertial or accelerating (including rotation) frame (reference frame traveling at constant velocity - including zero) to another is the Galilean transform.

Unprimed quantities refer to position, velocity and acceleration in one frame F; primed quantities refer to position, velocity and acceleration in another frame F' moving at translational velocity **V** or angular velocity **Ω** relative to F. Conversely F moves at velocity (—**V** or —**Ω**) relative to F'. The situation is similar for relative accelerations.

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| **Motion of entities** | **Inertial frames** | **Accelerating frames** |
| **Translation**  **V** = Constant relative velocity between two inertial frames F and F'. **A** = (Variable) relative acceleration between two accelerating frames F and F'. | Relative position  \mathbf{r}' = \mathbf{r} + \mathbf{V}t \,\!  Relative velocity  \mathbf{v}' = \mathbf{v} + \mathbf{V} \,\! Equivalent accelerations  \mathbf{a}' = \mathbf{a} | Relative accelerations  \mathbf{a}' = \mathbf{a} + \mathbf{A}  Apparent/fictitious forces  \mathbf{F}' = \mathbf{F} - \mathbf{F}_\mathrm{app} |
| **Rotation**  **Ω** = Constant relative angular velocity between two frames F and F'. **Λ** = (Variable) relative angular acceleration between two accelerating frames F and F'. | Relative angular position  \theta' = \theta + \Omega t \,\!  Relative velocity  \boldsymbol{\omega}' = \boldsymbol{\omega} + \boldsymbol{\Omega} \,\! Equivalent accelerations  \boldsymbol{\alpha}' = \boldsymbol{\alpha} | Relative accelerations  \boldsymbol{\alpha}' = \boldsymbol{\alpha} + \boldsymbol{\Lambda}  Apparent/fictitious torques  \boldsymbol{\tau}' = \boldsymbol{\tau} - \boldsymbol{\tau}_\mathrm{app} |
| Transformation of any vector **T** to a rotating frame  \frac{{\rm d}\mathbf{T}'}{{\rm d}t} = \frac{{\rm d}\mathbf{T}}{{\rm d}t} - \boldsymbol{\Omega} \times \mathbf{T} | |

**Mechanical oscillators**

SHM, DHM, SHO, and DHO refer to simple harmonic motion, damped harmonic motion, simple harmonic oscillator and damped harmonic oscillator respectively.

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| Equations of motion | | | |
| **Physical situation** | **Nomenclature** | **Translational equations** | **Angular equations** |
| **SHM** | * *x* = Transverse displacement * *θ* = Angular displacement * *A* = Transverse amplitude * Θ = Angular amplitude | \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = - \omega^2 x \,\!  Solution:  x = A \sin\left ( \omega t + \phi \right ) \,\! | \frac{\mathrm{d}^2 \theta}{\mathrm{d}t^2} = - \omega^2 \theta \,\!  Solution:  \theta = \Theta \sin\left ( \omega t + \phi \right ) \,\! |
| **Unforced DHM** | * *b* = damping constant * *κ* = torsion constant | \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + b \frac{\mathrm{d}x}{\mathrm{d}t} + \omega^2 x = 0 \,\!  Solution (see below for *ω'*): x=Ae^{-bt/2m}\cos\left ( \omega' \right )\,\!  Resonant frequency: \omega_\mathrm{res} = \sqrt{\omega^2 - \left ( \frac{b}{4m} \right )^2 } \,\!  Damping rate: \gamma = b/m \,\!  Expected lifetime of excitation: \tau = 1/\gamma\,\! | \frac{\mathrm{d}^2 \theta}{\mathrm{d}t^2} + b \frac{\mathrm{d}\theta}{\mathrm{d}t} + \omega^2 \theta = 0 \,\!  Solution: \theta=\Theta e^{-\kappa t/2m}\cos\left ( \omega \right )\,\!  Resonant frequency: \omega_\mathrm{res} = \sqrt{\omega^2 - \left ( \frac{\kappa}{4m} \right )^2 } \,\!  Damping rate: \gamma = \kappa/m \,\!  Expected lifetime of excitation: \tau = 1/\gamma\,\! |

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| Angular frequencies | | |
| **Physical situation** | **Nomenclature** | **Equations** |
| **Linear undamped unforced SHO** | * *k* = spring constant * *m* = mass of oscillating bob | \omega = \sqrt{\frac{k}{m}} \,\! |
| **Linear unforced DHO** | * *k* = spring constant * *b* = Damping coefficient | \omega' = \sqrt{\frac{k}{m}-\left ( \frac{b}{2m} \right )^2 } \,\! |
| **Low amplitude angular SHO** | * *I* = Moment of inertia about oscillating axis * *κ* = torsion constant | \omega = \sqrt{\frac{I}{\kappa}}\,\! |
| **Low amplitude simple pendulum** | * *L* = Length of pendulum * *g* = Gravitational acceleration * Θ = Angular amplitude | Approximate value  \omega = \sqrt{\frac{g}{L}}\,\!  Exact value can be shown to be: \omega = \sqrt{\frac{g}{L}} \left [ 1 + \sum_{k=1}^\infty \frac{\prod_{n=1}^k \left ( 2n-1 \right )}{\prod_{n=1}^m \left ( 2n \right )} \sin^{2n} \Theta \right ]\,\! |

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| Energy in mechanical oscillations | | |
| **Physical situation** | **Nomenclature** | **Equations** |
| **SHM energy** | * *T* = kinetic energy * *U* = potential energy * *E* = total energy | Potential energy  U = \frac{m}{2} \left ( x \right )^2 = \frac{m \left( \omega A \right )^2}{2} \cos^2(\omega t + \phi)\,\!Maximum value at x = A: U_\mathrm{max} \frac{m}{2} \left ( \omega A \right )^2  \,\!  Kinetic energy T = \frac{\omega^2 m}{2} \left ( \frac{\mathrm{d} x}{\mathrm{d} t} \right )^2 = \frac{m \left ( \omega A \right )^2}{2}\sin^2\left ( \omega t + \phi \right )\,\!  Total energy E = T + U \,\! |
| **DHM energy** |  | E = \frac{m \left ( \omega A \right )^2}{2}e^{-bt/m} \,\! |

**Notes**

* 1. [Mayer, Sussman & Wisdom 2001](http://en.wikipedia.org/wiki/List_of_equations_in_classical_mechanics#CITEREFMayerSussmanWisdom2001), p. xiii
  2. [Berkshire & Kibble 2004](http://en.wikipedia.org/wiki/List_of_equations_in_classical_mechanics#CITEREFBerkshireKibble2004), p. 1
  3. [Berkshire & Kibble 2004](http://en.wikipedia.org/wiki/List_of_equations_in_classical_mechanics#CITEREFBerkshireKibble2004), p. 2
  4. [Arnold 1989](http://en.wikipedia.org/wiki/List_of_equations_in_classical_mechanics#CITEREFArnold1989), p. v
  5. <http://www.ltcconline.net/greenl/courses/202/multipleIntegration/MassMoments.htm>, *Section: Moments and center of mass*
  6. R.P. Feynman, R.B. Leighton, M. Sands (1964). *Feynman's Lectures on Physics (volume 2)*. Addison-Wesley. pp. 31–7. [ISBN](http://en.wikipedia.org/wiki/International_Standard_Book_Number) [9-780-201-021172](http://en.wikipedia.org/wiki/Special:BookSources/9-780-201-021172).
  7. "Relativity, J.R. Forshaw 2009"
  8. "Mechanics, D. Kleppner 2010"
  9. "Relativity, J.R. Forshaw 2009"
  10. "Relativity, J.R. Forshaw 2009"

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* Arnold, Vladimir I. (1989), *Mathematical Methods of Classical Mechanics* (2nd ed.), Springer, [ISBN](http://en.wikipedia.org/wiki/International_Standard_Book_Number) [978-0-387-96890-2](http://en.wikipedia.org/wiki/Special:BookSources/978-0-387-96890-2)
* Berkshire, Frank H.; Kibble, T. W. B. (2004), *Classical Mechanics* (5th ed.), Imperial College Press, [ISBN](http://en.wikipedia.org/wiki/International_Standard_Book_Number) [978-1-86094-435-2](http://en.wikipedia.org/wiki/Special:BookSources/978-1-86094-435-2)
* Mayer, Meinhard E.; Sussman, Gerard J.; Wisdom, Jack (2001), *Structure and Interpretation of Classical Mechanics*, MIT Press, [ISBN](http://en.wikipedia.org/wiki/International_Standard_Book_Number) [978-0-262-19455-6](http://en.wikipedia.org/wiki/Special:BookSources/978-0-262-19455-6)

**External links**

* [Lectures on classical mechanics](http://www.astro.uvic.ca/%7Etatum/classmechs.html)
* [Biography of Isaac Newton, a key contributor to classical mechanics](http://scienceworld.wolfram.com/biography/Newton.html)

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